WORKSHOP CALCULATION & SCIENCE

(NSQF)

1st YEAR

(As per Revised Syllabus July 2022)

Civil Engineering Assistant

Directorate General of Training

DIRECTORATE GENERAL OF TRAINING
MINISTRY OF SKILL DEVELOPMENT & ENTREPRENEURSHIP
GOVERNMENTOF INDIA

NATIONAL INSTRUCTIONAL MEDIA INSTITUTE, CHENNAI

Post Box No. 3142, CTI Campus, Guindy, Chennai - 600 032

Workshop Calculation & Science

Civil Engineering Assistant - 1st Year NSQF

As per Revised Syllabus July 2022

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FOREWORD

The Government of India has set an ambitious target of imparting skills one out of every four Indians, to help them secure jobs as part of the National Skills Development Policy. Industrial Training Institutes (ITIs) play a vital role in this process especially in terms of providing skilled manpower. Keeping this in mind, and for providing the current industry relevant skill training to Trainees, ITI syllabus has been recently updated with the help of comprising various stakeholder's viz. Industries, Entrepreneurs, Academicians and representatives from ITIs.

The National Instructional Media Institute (NIMI), Chennai, has now come up with instructional material to suit the revised curriculum for **Workshop Calculation & Science - Civil Engineering Assistant 1st Year** NSQF (Revised 2022) under CTS will help the trainees to get an international equivalency standard where their skill proficiency and competency will be duly recognized across the globe and this will also increase the scope of recognition of prior learning. NSQF trainees will also get the opportunities to promote life long learning and skill development. I have no doubt that with NSQF the trainers and trainees of ITIs, and all stakeholders will derive maximum benefits from these IMPs and that NIMI's effort will go a long way in improving the quality of Vocational training in the country.

The Executive Director & Staff of NIMI and members of Media Development Committee deserve appreciation for their contribution in bringing out this publication.

Jai Hind

Ms. TRISHALJIT SETHI,
Additional Secretary / Director General (Training),
Ministry of Skill Development & Entrepreneurship,
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New Delhi - 110 001

PREFACE

The National Instructional Media Institute(NIMI) was set up at Chennai, by the Directorate General of Training, Ministry of skill Development and Entrepreneurship, Government of India, with the technical assistance from the Govt of the Federal Republic of Germany with the prime objective of developing and disseminating instructional Material for various trades as per prescribed syllabus and Craftsman Training Programme(CTS) under NSQF levels.

The Instructional materials are developed and produced in the form of Instructional Media Packages (IMPs), consisting of Trade Theory, Trade Practical, Test and Assignment Book, Instructor Guide. The above material will enable to achieve overall improvement in the standard of training in ITIs.

A national multi-skill programme called SKILL INDIA, was launched by the Government of India, through a Gazette Notification from the Ministry of Finance (Dept of Economic Affairs), Govt of India, dated 27th December 2013, with a view to create opportunities, space and scope for the development of talents of Indian Youth, and to develop those sectors under Skill Development.

The emphasis is to skill the Youth in such a manner to enable them to get employment and also improve Entrepreneurship by providing training, support and guidance for all occupation that were of traditional types. The training programme would be in the lines of International level, so that youths of our Country can get employed within the Country or Overseas employment. The **National Skill Qualification Framework (NSQF)**, anchored at the National Skill Development Agency(NSDA), is a Nationally Integrated Education and competency-based framework, to organize all qualifications according to a series of **levels of Knowledge, Skill and Aptitude.** Under NSQF the learner can acquire the Certification for Competency needed at any level through formal, non-formal or informal learning.

The **Workshop Calculation & Science -** Civil Engineering Assistant 1st Year NSQF (Revised 2022) under CTS is one of the book developed by the core group members as per the NSQF syllabus.

The **Workshop Calculation & Science -** Civil Engineering Assistant 1st Year NSQF (Revised 2022) under CTS as per NSQF is the outcome of the collective efforts of experts from Field Institutes of DGT, Champion ITI's for each of the Sectors, and also Media Development Committee **(MDC)** members and Staff of **NIMI.** NIMI wishes that the above material will fulfill to satisfy the long needs of the trainees and instructors and shall help the trainees for their Employability in Vocational Training.

NIMI would like to take this opportunity to convey sincere thanks to all the Members and Media Development Committee (MDC) members.

Chennai - 600 032 **EXECUTIVE DIRECTOR**

ACKNOWLEDGEMENT

The National Instructional Media Institute (NIMI) sincerely acknowledge with thanks the co-operation and contribution of the following Media Developers to bring this IMP for the course **Workshop Calculation & Science - Civil Engineering Assistant 1st Year** as per NSQF Revised 2022.

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Shri. G. Michael Johny	-	Manager, NIMI, Chennai - 32.

NIMI records its appreciation of the **Data Entry, CAD, DTP Operators** for their excellent and devoted services in the process of development of this IMP.

NIMI also acknowledges with thanks, the efforts rendered by all other staff who have contributed for the development of this book.

INTRODUCTION

The material has been divided into independent learning units, each consisting of a summary of the topic and an assignment part. The summary explains in a clear and easily understandable fashion the essence of the mathematical and scientific principles. This must not be treated as a replacment for the instructor's explanatory information to be imparted to the trainees in the classroom, which certainly will be more elaborate. The book should enable the trainees in grasping the essentials from the elaboration made by the instructor and will help them to solve independently the assignments of the respective chapters. It will also help them to solve the various problems, they may come across on the shop floor while doing their practical exercises.

The assignments are presented through 'Graphics' to ensure communications amongst the trainees. It also assists the trainees to determine the right approach to solve the problems. The required relevent data to solve the problems are provided adjacent to the graphics either by means of symbols or by means of words. The description of the symbols indicated in the problems has its reference in the relevant summaries.

At the end of the exercise wherever necessary assignments, problems are included for further practice.

Time allotment:

Duration of 1st Year : 40 Hrs

Time allotment for each title of exercises has been given below. **Workshop Calculation & Science - Civil Engineering Assistant** 1st Year NSQF Revised Syllabus 2022.

S.No	Title	Exercise No.	Time in Hrs
1	Unit, Fractions	1.1.01 - 1.1.07	4
2	Square root, Ratio and Proportions, Percentage	1.2.08 - 1.2.14	6
3	Material Science	1.3.15 - 1.3.19	6
4	Mass, Weight, Volume and Density	1.4.20	2
5	Heat & Temperature and Pressure	1.5.21 - 1.5.24	6
6	Basic Electricity	1.6.25 - 1.6.29	6
7	Mensuration	1.7.30 - 1.7.34	6
8	Trigonometry	1.8.35 - 1.8.38	4
		Total	**40 Hrs**

LEARNING / ASSESSABLE OUTCOME

On completion of this book you shall be able to

- **Demonstrate basic mathematical concept and principles to perform practical operations.**
- **Understand and explain basic science in the field of study.**

CONTENTS

SYLLABUS

1st Year

Workshop Calculation & Science - Civil Engineering Assistant
Revised syllabus July 2022 under CTS

S.No.	Title	Time in Hrs
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II	**Square root, Ratio and Proportions, Percentage** 1 Square and Square root 2 Simple problems using calculator 3 Applications of Pythagoras theorem and related problems 4 Ratio and Proportion 5 Ratio and Proportion - Direct and Indirect proportions 6 Percentage 7 Percentage - Changing percentage to decimal and fraction	6
III	**Material Science** 1 Types of metal, types of ferrous and non ferrous metals 2 Physical and Mechanical Properties of metals 3 Introduction of iron and cast iron 4 Difference between iron & steel, alloy steel and carbon steel 5 Properties and uses of timber	6
IV	**Mass, Weight, Volume and Density** 1 Mass, volume, density, weight and specific gravity	2
V	**Heat & Temperature and Pressure** 1 Concept of heat and temperature, effects of heat, difference between heat and temperature, boiling point & melting point of different metals and non-metals 2 Scales of temperature, Celsius, Fahrenheit, Kelvin and Conversion between scales of temperature	6

S.No.	Title	Time in Hrs
	3 Temperature measuring instruments, types of thermometer, pyrometer and transmission of heat - Conduction, convection and radiation 4 Co-efficient of linear expansion and related problems with assignments	
VI	**Basic Electricity** 1 Introduction and uses of electricity, molecule, atom, how electricity is produced, electric current AC, DC their comparison, voltage, resistance and their units 2 Conductor, Insulator, types of connections - Series and Parallel 3 Ohm's Law, relation between VIR & related problems 4 Electrical power, energy and their units, calculation with assignments 5 Electrical Power, HP, Energy and units of electrical energy	6
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VIII	**Trigonometry** 1 Measurement of angles 2 Trigonometrical ratios 3 Trigonometrical tables 4 Application in calculating height and distance (Simple applications)	4
	Total	40

Unit, Fractions - Classification of unit system

Necessity

All physical quantities are to be measured in terms of standard quantities.

Unit

A unit is defined as a standard or fixed quantity of one kind used to measure other quantities of the same kind.

Classification

Fundamental units and derived units are the two classifications.

Fundamental units

Units of basic quantities of length, mass and time.

Derived units

Units which are derived from basic units and bear a constant relationship with the fundamental units.E.g. area, volume, pressure, force etc.

Systems of units

- F.P.S system is the British system in which the basic units of length, mass and time are foot, pound and second respectively.
- C.G.S system is the metric system in which the basic units of length, mass and time are centimeter, gram and seconds respectively.
- M.K.S system is another metric system in which the basic units of length, mass and time are metre, kilogram and second respectively.
- S.I. units are referred to as Systems International units which is again of metric and the basic units, their names and symbols are as follows.

Fundamental units and derived units are the two classifications of units.

Length, mass and time are the fundamental units in all the systems (i.e) F.P.S, C.G.S, M.K.S and S.I. systems.

Example

Length: What is the length of copper wire in the roll , if the roll of copper wire weighs 8kg, the dia of wire is 0.9cm and the density is 8.9 gm/cm^3?

Solution

mass of copper wire in the roll = 8kg (or)8000grams
Dia of copper wire in the roll = 0.9cm
Density of copper wire = 8.9 gm/cm^3

Area of cross section of copper wire

$$= \frac{\pi d^2}{4} = \frac{\pi \times (0.9^2)}{4} = 0.636 cm^2$$

Volume of copper wire

$$= \frac{\text{Mass of copper wire}}{\text{Density of copper wire}} = \frac{8000 grams}{8.9\ gm/cm^3} = 898.88 cm^3$$

Length of copper wire

$$= \frac{\text{Volume of copper wire}}{\text{Area of cross section of copper wire}} = \frac{898.88 cm^3}{0.636 cm^2}$$

$$= 1413.33 \text{ cm}$$

Length of copper wire =1413cm.

Time: The S.I. unit of time, the second, is another base units of S.I., it is defined as the time interval occupied by a number of cycles of radiation from the calcium atom. The second is the same quantity in the S.I. in the British and in the U.S. systems of units.

Fundamental units of F.P.S, C.G.S, M.K.S and S.I

S.No.	Basic quantity	British units		Metric units				International units	
		F.P.S	Symbol	C.G.S	Symbol	M.K.S	Symbol	S.I Units	Symbol
1	Length	Foot	ft	Centimetre	cm	Metre	m	Metre	m
2	Mass	Pound	lb	Gram	g	Kilogram	kg	Kilogram	Kg
3	Time	Second	s	Second	s	Second	s	Second	s
4	Current	Ampere	A	Ampere	A	Ampere	A	Ampere	A
5	Temperature	Fahrenheit	°F	Centigrade	°C	Centigrade	°C	Kelvin	K
6	Light intensity	Candela	Cd	Candela	Cd	Candela	Cd	Candela	Cd

Unit, Fractions - Fundamental and Derived units F.P.S, C.G.S, M.K.S and SI units

Derived units of F.P.S, C.G.S, M.K.S and SI system

S.No	Physical quantity	British units		Metric units				International units	
		FPS	Symbol	CGS	Symbol	MKS	Symbol	SI Units	Symbol
1	Area	Square foot	ft^2	Square centimetre	cm^2	Square metre	m^2	Square metre	m^2
2	Volume	Cubic foot	ft^3	Cubic centimetre	cm^3	Cubic metre	m^3	Cubic metre	m^3
3	Density	Pound per cubic foot	lb/ft^3	Gram per cubic centimetre	g/cm^3	Kilogram per cubic metre	kg/m^3	Kilogram per cubic metre	Kg/m^3
4	Speed	Foot per second	ft/s	Centimetre per second	cm/sec	Metre per second	m/sec	Metre per second	m/sec
5	Velocity (linear)	Foot per second	ft/s	Centimetre per second	cm/sec	Metre per second	m/sec	Metre per second	m/sec
6	Acceleration	Foot per square second	ft/s^2	Centimetre per square second	cm/sec^2	Metre per square second	m/sec^2	Metre per square second	m/sec^2
7	Retardation	Foot per square Second	ft/s^2	Centimetre per square second	cm/sec^2	Metre per square second	m/sec^2	Metre square second	m/sec^2
8	Angular velocity	Degree per second	Deg/sec	Radian per second	rad/sec	Radian per second	rad/sec	Radian per second	rad/sec
9	Mass	Pound (slug)	lb	Gram	g	Kilogram	kg	Kilogram	kg
10	Weight	Pound	lb	Gram	g	Kilogram weight	kg	Newton	N
11	Force	Pounds	lbf	dyne	dyn	Kilogram force	kgf	Newton	$N(kgm/sec^2)$
12	Power	Foot pound per second	ft.lb/sec	Gram.centimetre/sec	g.cm/ sec	kilogram metre per second	kg.m/ sec	-	-
		Horse power	hp	Erg per second		watt	W	watt	W(J/sec)
13	Pressure,Stress	Pound per square inch	lb/in^2	Gram per square centimetre	g/cm^2	Kilogram per square metre	kg/m^2	Newton per square metre	N/m^2
14	Energy, Work	Foot.pound	ft.lb	Gram centimetre	g.cm	Kilogram metre	kg.m	joule	J(Nm)
15	Heat	British thermal unit	BTU	calorie	Cal	joule	J	joule	J(Nm)
16	Torque	Pound force foot	lbf.ft	Newton millimetre	N mm	Kilogram metre	kg.m	Newton metre	Nm
17	Temperature	Degree Fahrenheit	°F	Degree Centigrade	°C	Kelvin	K	Kelvin	K

Unit, Fractions - Measurement units and conversion

Units and abbreviations

Quantity	Units	Abbreviation of unit
Calorific value	kilojoules per kilogram	kJ/kg
Specific fuel consumption	kilogram per hour per newton	kg/hr/N
Length	millimetre, metre, kilometre	mm, m, km
Mass	kilogram, gram	kg, g
Time	seconds, minutes, hours	s, min, h
Speed	centimetre per second, metre per second kilometre per hour, miles per hour	cm/s, m/s km/h, mph
Acceleration	metre-per-square second	m/s^2
Force	newtons, kilonewtons	N,kN
Moment	newton-metres	Nm
Work	joules	J
Power	horsepower, watts, kilowatts	Hp, W, kW
Pressure	newton per square metre kilonewton per square metre	N/m^2 kN/m^2
Angle	radian	rad
Angular speed	radians per second radians-per-square second revolutions per minute revolutions per second	rad/s rad/s^2 Rpm rev/s

Decimal multiples and parts of unit

Decimal power	Value	Prefixes	Symbol	Stands for
10^{12}	1000000000000	tera	T	billion times
10^{9}	1000000000	giga	G	thousand millintimes
10^{6}	1000000	mega	M	million times
10^{3}	1000	kilo	K	thousand times
10^{2}	100	hecto	h	hundred times
10^{1}	10	deca	da	ten times
10^{-1}	0.1	deci	d	tenth
10^{-2}	0.01	centi	c	hundredth
10^{-3}	0.001	milli	m	thousandth
10^{-6}	0.000001	micro	μ	millionth
10^{-9}	0.000000001	nano	n	thousand millionth
10^{-12}	0.000000000001	pico	p	billionth

SI units and the British units:

Quantity	SI unit → British unit	British unit → SI unit
Length	1 m = 3.281 ft 1 km = 0.621 mile	1 ft = 0.3048 m 1 mile = 1.609 km
Speed	1 m/s = 3.281 ft/s 1 km/h = 0.621 mph	1 ft/s = 0.305 m/s 1 mph = 1.61 km/h
Acceleration	1 m/s^2 = 3.281 ft/s^2	1 ft/s^2 = 0.305 m/s^2
Mass	1 kg = 2.205 lb	1 lb = 0.454 kg
Force	1 N = 0.225 lbf 1 MN	1 lbf = 4.448 N 1 million newtons
Torque	1 Nm = 0.738 lbf ft	1 lbf ft = 1.355 Nm
Pressure	1 N/m^2 = 0.000145 lbf/in^2 1 Pa = 1 N/m^2 1 bar = 14.5038 lbf/in^2	1 lbf/in^2 = 6.896 kN/m^2 1 lbf/in^2 = 6.895 kN/m^2
Energy, work	1 J = 0.738 ft lbf 1 J = 0.239 calorie 1 kJ = 0.948 BTU (1 therm = 100 000 BTU) 1 kJ = 0.526 CHU	1 ft lbf = 1.355 J 1 calorie = 4.186 J 1 BTU = 1.055 kJ 1 CHU = 1.9 kJ
Power	1 kW = 1.34 hp	1 hp = 0.7457 kW
Fuel consumption	1km/L = 2.82 mile/gallon	1 mpg = 0.354 km/L
Specific fuel consumption	1 kg/kWh = 1.65 lb/bhp h 1 litre/kWh=1.575 pt/bhp h	1 lb/bhp h = 0.606 kg/kWh 1 pt/bhp h = 0.631 litre/kWh
Calorific value	1 kJ/kg = 0.43 BTU/lb 1 kJ/kg = 0.239 CHU/lb	1 BTU/lb = 2.326 kJ/kg 1 CHU/lb = 4.188 kJ/kg

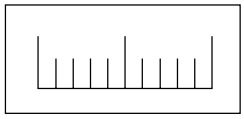
Prefixes for decimal multiples and submultiples

Use		
1 Megapascal	= 1 MPa	= 1000000 Pa
1 Kilowatt	= 1 kW	= 1000 W
1 Hectolitre	= 1 hL	= 100 L
Decanewton	= 1 daN	= 10 N
Decimetre	= 1 dm	= 0.1 m
1 Centimetre	= 1 cm	= 0.01 m
1 Millimetre	= 1 mm	= 0.001 m
1 Micrometre	= 1 um	= 0.000001 m

Conversion factors

1 inch	=	25.4 mm
1 mm	=	0.03937 inch
1 metre	=	39.37 inch
1 micron	=	0.00003937"
1 kilometre	=	0.621 miles
1 pound	=	453.6 g
1 kg	=	2.205 lbs
1 metric ton	=	0.98 ton

Units of physical quantities

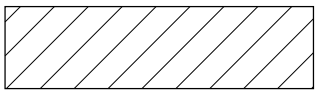


UNITS OF LENGTH

Units of length

Micron	1 μ	=	0.001 mm
Millimetre	1 mm	=	1000 μ
Centimetre	1 cm	=	10 mm
Decimetre	1 dm	=	10 cm
Metre	1 m	=	10 dm
Kilometre	1 km	=	1000 m
Inch	1"	=	25.4 mm
Foot	1’	=	0.305 m
Yard	1 Yd	=	0.914 m
Nautical mile	1 NM	=	1852 m
Geographical mile	1	=	1855.4 m

Units of area



UNITS OF AREA

Square millimetre	1 mm^2	
Square centimetre	1 cm^2	= 100 mm^2
Square decimetre	1 dm^2	= 100 cm^2
Square metre	1 m^2	= 100 dm^2
Are	1 a	= 100 m^2
Hectare	1 ha	= 100 a
Square kilometre	1 km^2	= 100 ha
Square inch	1 sq.in	= 6.45 cm^2
Square foot	1 sq.ft	= 0.093 m^2
Square yard	1 sq.yd	= 0.84 m^2
Square metre	1 m^2	= 10.76 ft^2
Acre	1	= 40.5 a

1 Acre = 100 cent

1 Cent = 436 Sq. ft.
1 Ground = 2400 Sq.ft.

1 Hectare = 2.47 acres

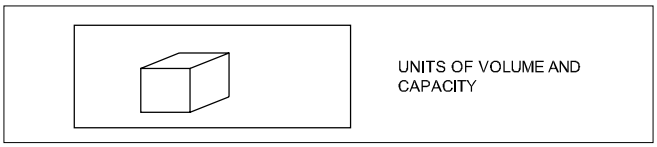
1 acre = 0.4047 Hectare

1 Hectare = 10000 sq. metre

Units of weight

UNITS OF WEIGHT

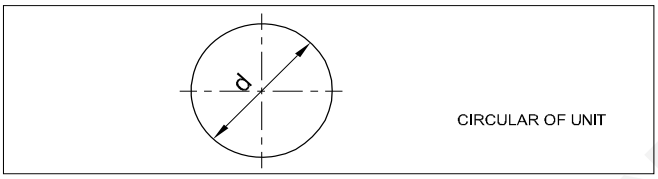
Milligram - force	1 mgf	
Gram-force	1 gf	1000 mgf
Kilogram-force	1 kgf	= 1000 gf
Tonne	1 t	= 1000 kgf
Ounce	1	= 28.35 gf
Pound	1 lbs	= 0.454 kgf
Long ton	1	= 1016 kgf
Short ton	1	= 907 kgf



Units of volume and capacity

Cubic millimetre	1 mm^3	
Cubic centimetre	1 cm^3	= 1000 mm^3
Cubic decimetre	1 dm^3	= 1000 cm^3
Cubic metre	1 m^3	= 1000 dm^3
Litre	1 l	= 1 dm^3
Hectolitre	1 hl	= 100 l
Cubic inch	1 cu. in	= 16.387 cm^3
Cubic foot	1 cu. ft	= 28317 cm^3
Gallon (British)	1 gal	= 4.54 l
1cubic metre	1 m^3	= 1000 litres
1000 Cu.cm	1000 cm^3	= 1 litre
1 cubic foot	1 ft^3	= 6.25 Gallon
1 litre	1lt	= 0.22 Gallon

Circular unit



Radian

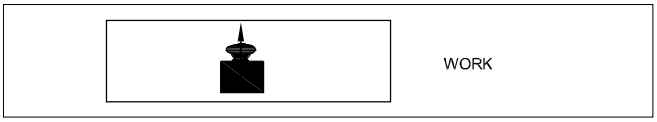
Relationship between Radian and Degree

1 Radian = $\frac{180^\circ}{\pi}$

180^0 = π Radian;

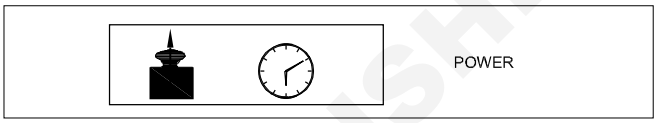
1 Degree = $\frac{\pi}{180}$ Radian

Work



Kilogram-force	1 kgfm	= 9.80665 J
Metre	1 kgfm	= 9.80665 Ws
Joule	1 J	= 1 Nm
Watt-second	1 Ws	= 0.102 kgfm
Kilowatt hour	1 kWh	= 3.6 x 10^6 J
		= 859.8456 $kcal_{IT}$
I.T.Kilocalorie	1 $kcal_{IT}$	= 426.kgfm

Power



Kilogram-force metre/second

1 kgfm/s	= 9.80665 W
Kilowatt	1 kW = 1000 W = 1000 J/s
	= 102 kgfm/s (approx.)
Metric horse power	1 HP = 75 kgfm/s
	= 0.736 kW
1 Calorie	= 4.187J

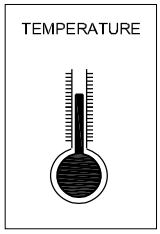
I.T.Kilocalorie/hour = 1 $kcal_{IT/h}$ = 1.163 W

Pressure

Pascal	1 Pa	= 1 N/m^2	1 atm	= 101325 Pa
Bar	1 bar = 10N/cm^2	= 100000 Pa–Torr	1 torr	= $\frac{101325}{760}$ ≈ 133.32 pa
Atmosphere	1 atm	= 1 kgf/cm^2	1 kgf/cm^2 = 735.6 mm of mercury	

TEMPERATURE

Scale	Freezing point	Boiling point
Centigrade (°C)	0°C	100°C
Fahrenheit (°F)	32°F	212°F
Kelvin (K)	273K	373K
Reaumur (°R)	0°R	80°R



$$\frac{^\circ R}{80} = \frac{^\circ C}{100} = \frac{K - 273}{100} = \frac{^\circ F - 32}{180}$$

FORCE

Force	In C.G.S. System : Force (Dyne)	= Mass (gm)XAcceleration (cm/sec^2)
	In F.P.S. System : Force (Poundal)	= Mass (lb) X Acceleration (ft./sec^2)
	In M.K.S System : Force (Newton)	= Mass (Kg) x Acceleration (mtr./sec^2)
	1 Dyne	= 1 gm x1 cm/sec^2
	1 Poundal	= 1 lb x 1 ft/sec^2
	1 Newton	= 1 kg x 1 mtr/sec^2 = 10^5 dynes
	1gm weight	= 981 Dynes
	1 lb weight	= 32 Poundals
	1 kg weight	= 9.81 Newtons

ELECTRICAL QUANTITIES

V	Electric potential	V	Volt	V(W/A)
E	Electromotive force	V	Volt	V(W/A)
I	Electric current	A	Ampere	A
R	Electric resistance	Ω	Ohm	Ω (V/A)
e	Specific resistance	Ω m	Ohm metre	Vm/A
G	Conductance	Ω^{-1}	Siemens	S

Assignment - Answer the following question.

1 Convert 320 kilometres into miles _____

2 Convert 16 tons into kilograms _____

3 Convert 40 inches into centimetres _____

4 Convert 8 metres into feet _____

5 Convert 2.5 gallons into litres _____

6 Convert 5 litres into gallons _____

7 120°C = _____ °F.

8 Expand the abbreviations of the following

a N/m^2

b RPM

9 Convert the following S.I. units as required.

a Length

i 3.4 m = _____ mm

ii 10.2 km = _____ mile

b Mass

i 650 g = _____ kg

ii 120 mg = _____ g

c Force

i 1.2 N = _____ kg

ii 25 kg = _____ N

d Work, energy, amount of heat

i 120 KJ = _____ J

ii 300 wh = _____ kwh

e Power

i 0.2 kW = _____ W

ii 350 W = _____ kW

f Convert as required.

i 5 N = _____ KN

Unit, Fractions - Factors, HCF, LCM and problems

Prime Numbers and whole Numbers

Factor

A factor is a small number which divides exactly into a bigger number.e.g.

To find the factors of 24, 72, 100 numbers

24 = 2 x 2 x 2 x 3

72 = 2 x 2 x 2 x 3 x 3

100 = 2 x 2 x 5 x 5

The numbers 2,3,5 are called factors.

Definition of a prime factor

Prime factor is a number which divides a prime number into factors.e.g.

57 = 3 x 19

The numbers 3 and 19 are prime factors.

They are called as such, since 3 & 19 also belong to prime number category.

Definition of H.C.F

The Highest Common Factor

The H.C.F of a given group of numbers is the highest number which will exactly divide all the numbers of that group.e.g.

To find the H.C.F of the numbers 24, 72, 100

24 = 2 x 2 x 2 x 3

72 = 2 x 2 x 2 x 3 x 3

100 = 2 x 2 x 5 x 5

The factors common to all the three numbers are

2 x 2 = 4. So HCF = 4.

Definition of L.C.M

Lowest common multiple

The lowest common multiple of a group of numbers is the smallest number that will contain each number of the given group without a remainder.e.g.

- Factorise the following numbers

  7,17,20,66,128

  7,17 - These two belong to Prime numbers. Hence no factor except unity and itself.

2	20
2	10
	5

Factors of 20 = 2 x 2 x 5

2	66
3	33
	11

Factors of 66 = 2 x 3 x 11

2	128
2	64
2	32
2	16
2	8
2	4
	2

Factors of 128 = 2 x 2 x 2 x 2 x 2 x 2 x 2

- Select prime numbers from 3 to 29

  3,5,7,11,13,17,19,23,29

- Find the HCF of the following group of numbers HCF of 78, 128, 196

2	78
3	39
	13

78 = 2 x 3 x 13

2	128
2	64
2	32
2	16
2	8
2	4
	2

128 = 2 x 2 x 2 x 2 x 2 x 2 x 2

2	196
2	98
	49

196 = 2 x 2 x 49

HCF = 2

- Find LCM of 84,92,76

LCM =

2	84, 92, 76
2	42, 46, 38
3	21, 23, 19
	7, 23, 19

LCM = 2 x 2 x 3 x 7 x 23 x 19 = 36708

- To find out the LCM of 36, 108, 60

2	36, 108, 60
2	18, 54, 30
3	9, 27, 15
3	3, 9, 5
	1, 3, 5

LCM of the number

36, 108, 60 = 2 x 2 x 3 x 3 x 3 x 5 = 540

The necessity of finding LCM and HCF arises in subtraction and addition of fractions.

Unit, Fractions - Fractions - Addition, subtraction, multiplication & division

Description

A minimal quantity that is not a whole number. For e.g. .

$\frac{1}{5}$ a vulgur fraction consists of a numerator and denominator.

Numerator/Denominator

The number above the line in a vulgar fraction showing how many of the parts indicated by the denominator are taken is the numerator. The total number of parts into which the whole quantity is divided and written below the line in a vulgar fraction is the denominator. e.g.

$\frac{1}{4}, \frac{3}{4}, \frac{7}{12}$

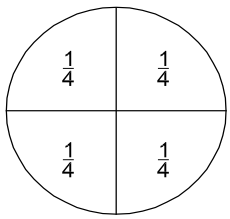
1,3,7 - numerators 4,12 - denominators

Fraction: Concept

Every number can be represented as a fraction.e.g.

$1\frac{1}{4} = \frac{5}{4}$, A full number can be represented as an apparent fraction.e.g. (Fig 1)

Fig 1



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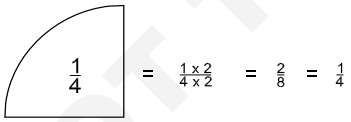
Fraction: Value

The value of a fraction remains the same if the numerator and denominator of the fraction are multiplied or divided by the same number.(Fig 2)

Fig 2



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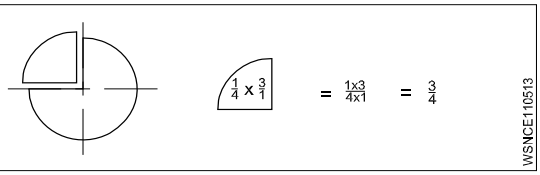
Multiplication

When fractions are to be multiplied, multiply all the numerators to get the numerator of the product and multiply all the denominators to form the denominator of the product. (Fig 3)

Fig 3



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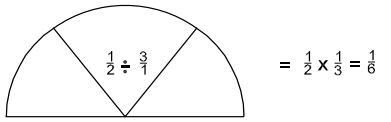
Division

When a fraction is divided by another fraction the dividend is multiplied by the reciprocal of the divisor. (Fig 4)

Fig 4



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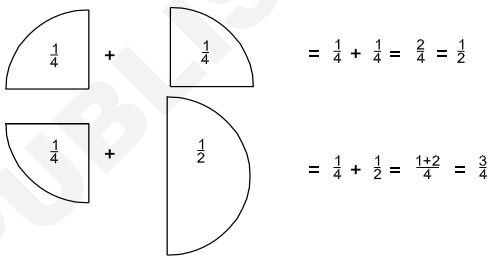
Addition and Subtraction

The denominators of the fractions should be the same when adding or subtracting the fractions. Unequal denominators must first be formed into a common denominator. It is the lowest common denominator and it is equal to the product of the most common prime numbers of the denominators of the fractions in question.(Fig 5)

Fig 5



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Examples

- Multiply $\frac{3}{4}$ by $\frac{2}{3}$,

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$

- Divide $\frac{3}{8}$ by $\frac{3}{4}$,

$$\frac{3}{8} \div \frac{3}{4} = \frac{3}{8} \text{x} \frac{4}{3} = \frac{1}{2}$$

- Add $\frac{3}{4}$ and $\frac{2}{3}$,

$$\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$$

- sub $\frac{7}{16}$ from $\frac{17}{32}$

$$\frac{17}{32} - \frac{7}{16} = \frac{17}{32} - \frac{14}{32} = \frac{(17-14)}{32} = \frac{3}{32}$$

Types of fractions

- Proper fractions are less than unity. Improper fractions have their numerators greater than the denominators.
- A mixed number has a full number and a fraction.

Addition of fraction

$\text{Add}\frac{1}{2}+\frac{1}{8}+\frac{5}{12}$

To add these fractions we have to find out L.C.M of denominators 2,8,12.

Find L.C.M of 2,8,12

Step 1 L.C.M

2	2,8,12
2	1,4,6
	1,2,3

Factors are 2,2,2,3

Hence L.C.M = 2 x 2 x 2 x 3 = 24

Step 2

$$\frac{1}{2}+\frac{1}{8}+\frac{5}{12}=\frac{12}{24}+\frac{3}{24}+\frac{10}{24}$$

$$=\frac{12+3+10}{24}=\frac{25}{24}=1\frac{1}{24}.$$

Subtraction of fraction

$\text{subtract } 9\frac{15}{32}\text{from}17\frac{9}{16}\text{or}(17\frac{9}{16}-9\frac{15}{32})$

Step 1: Subtract whole number first 17 - 9 = 8

Step 2: L.C.M of 16,32 = 32

Since number 16 divides the number 32

Subtracting fractions = $\frac{3}{32}$

Adding with whole number from Step 1

we get $8+\frac{3}{32}=8\frac{3}{32}$

Common fractions

Problems with plus and minus sign

Example

$\text{solve } 3\frac{3}{4}+6\frac{7}{8}-4\frac{5}{16}-\frac{9}{32}$

Rule to be followed

1 Add all whole numbers

2 add all + Numbers

3 Add all - Numbers

4 Find L.C.M of all denominators

Solution

Step 1: Add whole numbers = 3 + 6 - 4 = 5

Step 2: Add fractions = $\frac{3}{4}+\frac{7}{8}-\frac{5}{16}-\frac{9}{32}$

L.C.M of 4,8,16,32 is 32

$$\frac{24+28-10-9}{32}$$

$$=\frac{52-19}{32}$$

$$=\frac{33}{32}=1\frac{1}{32}$$

Step 3: Adding again with the whole number

we get $5+1\frac{3}{32}=6\frac{3}{32}$

Examples

Common fractions

- Multiply

a $\frac{3}{8}\text{by}\frac{4}{7}=\frac{3}{8}\text{x}\frac{4}{7}=\frac{3}{14}$ b $\frac{2}{3}\text{x}\frac{3}{4}\text{x}\frac{5}{8}=\frac{5}{16}$

- Division

a $\frac{5}{16}\div\frac{5}{32}=\frac{5}{16}\text{x}\frac{32}{5}=2$

b $4\frac{2}{3}\div3\frac{1}{7}=\frac{14}{3}\div\frac{22}{7}=\frac{14}{3}\text{x}\frac{7}{22}=\frac{49}{33}=1\frac{16}{33}$

- Addition

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$

L..C.M $= 2,4,8 = 8$

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=\frac{4+2+1}{8}=\frac{7}{8}$

- Subtraction

$$5\frac{1}{4}-3\frac{3}{4}=5-3+\frac{1}{4}-\frac{3}{4}$$

$$=2+\frac{1}{4}-\frac{3}{4}=2\frac{1}{4}-\frac{3}{4}$$

$$=\frac{9}{4}-\frac{3}{4}=\frac{9-3}{4}$$

$$=\frac{6}{4}=\frac{3}{2}=1\frac{1}{2}$$

Assignment

1 Convert the following into improper fractions.

a $1\frac{2}{7} =$ _____

b $4\frac{3}{5} =$ _____

c $3\frac{3}{5} =$ _____

2 Convert the following into mixed numbers.

a $\frac{12}{11} =$ _____

b $\frac{36}{14} =$ _____

c $\frac{18}{10} =$ _____

3 Place the missing numbers.

a $\frac{11}{13} = \frac{x}{91}$ _____

b $\frac{3}{5} = \frac{42}{x}$ _____

c $\frac{9}{14} = \frac{x}{98}$ _____

4 Simplify.

a $\frac{45}{60} =$ _____

b $\frac{8}{12} =$ _____

5 Multiply.

a $5 \times \frac{2}{3} =$ _____

b $\frac{3}{4} \times 2 =$ _____

c $\frac{3}{4} \times \frac{5}{6} =$ _____

6 Divide

a $\frac{1}{4} \div \frac{3}{4} =$ _____

b $6 \div \frac{3}{4} =$ _____

c $\frac{3}{4} \div \frac{2}{7} =$ _____

7 Place the missing numbers.

a $\frac{2}{3} = \frac{1}{12} x$_____

b $\frac{14}{24} = \frac{1}{12} x$_____

c $\frac{7}{8} = \frac{1}{12} x$_____

8 Add the followings:

a $\frac{3}{4} + \frac{7}{12} =$ _____

b $\frac{7}{8} + \frac{3}{4} =$ _____

9 Subtract

a $\frac{4}{5} - \frac{2}{5} =$ _____

b $\frac{5}{6} - \frac{3}{4} =$ _____

10 Simplify

a $2\frac{6}{7} - \frac{3}{8} - \frac{1}{3} - 1\frac{1}{16} =$ _____

b $2\frac{2}{7} - \frac{5}{6} + 8 =$ _____

11 Express as improper fractions

a $5\frac{3}{4}$

b $3\frac{5}{64}$

c $1\frac{5}{12}$

Unit, Fractions - Decimal fractions - Addition, subtraction, multiplication & division

Description

Decimal fraction is a fraction whose denominator is 10 or powers of 10 or multiples of 10 (i.e.) 10, 100, 1000, 10000 etc. Meaning of a decimal number:-

12.3256 means

$$(1 \text{ X } 10) + (2 \text{ X } 1) + \frac{3}{10} + \frac{2}{100} + \frac{5}{1000} + \frac{6}{10000}$$

Representation

The denominator is omitted. A decimal point is placed at different positions of the number corresponding to the magnitude of the denominator

$$Ex. \frac{5}{10} = 0.5, \frac{35}{100} = 0.35 \frac{127}{10000} = 0.0127, \frac{3648}{1000} = 3.648$$

Addition and subtraction

Arrange the decimal fractions in a vertical order, placing the decimal point of each fraction to be added or subtracted, in succession one below the other, so that all the decimal points are arranged in a straight line. Add or subtract as you would do for a whole number and place the decimal point in the answer below the column of decimal points.

Decimal fractions less than 1 are written with a zero before the decimal point. Example: 45/100 = 0.45 (and not simply .45)

Add 0.375 + 3.686

$$\begin{array}{r} 0.375 \\ 3.686 \\ \hline 4.061 \end{array}$$

Subtract 18.72 from 22.61

$$\begin{array}{r} 22.61 \\ 18.72 \\ \hline 3.89 \end{array}$$

Multiplication

Ignore the decimal points and multiply as whole numbers. Find the total number of digits to the right of the decimal point. Insert the decimal point in the answer such that the number of digits to the right of the decimal point equals to the sum of the digits found to the right of the decimal points in the problem.

Multiply 2.5 by 1.25

= 25 x 125 = 3125. The sum of the figures to the right of decimal point is 3. Hence the answer is 3.125.

Division

Move the decimal point of the divisor to the right to make it a full number. Move the decimal point in the dividend to the same number of places, adding zeroes if necessary. Then divide.

Divide 0.75 by 0.25

$$0.25\overline{)0.75}$$

$$\frac{0.75}{0.25} \text{ x } \frac{100}{100} = \frac{75}{25}$$

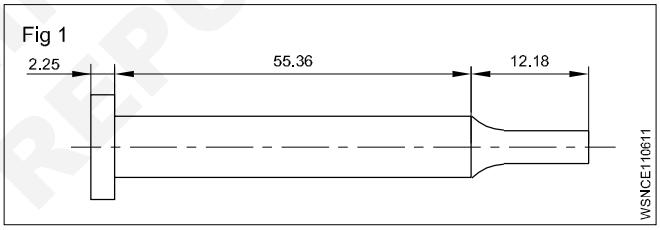
$$25\overline{)75} = 3$$

Move the decimal point in the multiplicand to the right to one place if the multiplier is 10, and to two places if the multiplier is 100 and so on. When dividing by 10 move the decimal point one place to the left, and, if it is by 100, move them point by two places and so on.

Example

Allowance allowing 3 mm for cutting off each pin, how many pins can be made from a 900 mm long bar and how much material will be left out?

Fig 1



Total Length of pin = 2.25 + 55.36 + 12.18

= 69.79 mm

Cutting allowance = 3 mm

Total Length = length of pin + cutting allowance

= 69.79 mm + 3 mm

= 72.79 mm

Length of the bar = 900 mm

No.of pins to be cut = $\frac{900}{72.79}$ = 12.394

= 12 pins

Left out material = Total length - length of pin + cutting allowance

= 900 - 12 x 69.79 + 12 x 3

= 900 - 837.48 + 36

= 900 - 873.48

Left out length = 26.52 mm

Conversion of Decimals into fractions and vice-versa

- Convert decimal into fractions

Example

Convert 0.375 to a fraction

Now place 1 under the decimal point followed by as many zeros as there are numbers

$$0.375 = \frac{375}{1000} = \frac{15}{40} = \frac{3}{8}$$

$$0.375 = \frac{3}{8}$$

- Convert fraction into decimal

Example

- Convert $\frac{9}{16}$ to a decimal

Proceed to divide $\frac{9}{16}$ in the normal way of division but put zeros (as required) after the number 9 (Numerator)

$$\begin{array}{r} 0.5625 \\ 16\overline{)90000} \\ 80 \\ \hline 100 \\ 96 \\ \hline 40 \\ 32 \\ \hline 80 \\ 80 \\ \hline 0 \end{array}$$

$\frac{9}{16} = 0.5625$

Recurring decimals

While converting from fraction to decimals, some fractions can be divided exactly into a decimal. In some fractions the quotient will not stop. It will continue and keep recurring. These are called recurring decimals.

Examples

- convert $\frac{1}{3}, \frac{2}{3}, \frac{1}{7}$

a $\frac{1}{3} = \frac{10000}{3} = 0.3333 - \text{Recurring}$

b $\frac{2}{3} = \frac{20000}{3} = 0.666 - \text{Recurring}$

c $\left(\frac{1}{7} = \frac{10000}{7} = 0.142857142 - \text{Recurring}\right)$

Method of writing approximations in decimals

1.73556 = 1.7356 Correct to 4 decimal places
5.7343 = 5.734 Correct to 3 decimal places
0.9345 = 0.94 Correct to 2 decimal places

Multiplication and division by 10,100,1000

Multiplying decimals by 10

A decimal fraction can be multiplied by 10,100,1000 and so on by moving the decimal point to the right by as many places as there are zeros in the multiplier.

- 4.645 x 10 = 46.45 (one place)
- 4.645 x 100 = 464.5 (two places)
- 4.645 x 1000 = 4645 (three places)

Dividing decimals by 10

A decimal fraction can be divided by 10,100,1000 and so on, by moving the decimal point to the left by as many places as required in the divisor by putting zeros

Examples

- 3.732 ÷ 10 = 0.3732 (one place)
- 3.732 ÷ 100 = 0.03732 (two places)
- 3.732 ÷ 1000 = 0.003732 (three places)

Examples

- Rewrite the following number as a fraction

  453.273

$$= (4 \times 100) + (5 \times 10) + (3 \times 1) + \frac{2}{10} + \frac{7}{100} + \frac{3}{100}$$

$$= 453\frac{273}{1000}$$

- Write the representation of decimal places in the given number 0.386

  3 - Ist decimal place
  8 - IInd decimal place
  6 - IIIrd decimal place

- Write approximations in the following decimals to 3 places.

  a 6.9453 ——> 6.945
  b 8.7456 ——> 8.746

- Convert fraction to decimal

$$\frac{21}{24} = \frac{7}{8} = 0.875$$

- Convert decimal to fraction

$$0.0625 = \frac{625}{10000} = \frac{5}{80} = \frac{1}{16}$$

Assignment

1 Write down the following decimal numbers in the expanded form.

a 514.726

b 902.524

2 Write the following decimal numbers from the expansion.

a $500 + 70 + 5 + \frac{3}{10} + \frac{2}{100} + \frac{9}{1000}$

b $200 + 9 + \frac{1}{10} + \frac{3}{100} + \frac{5}{1000}$

3 Convert the following decimals into fractions in the simplest form.

a 0.72

b 5.45

c 3.64

d 2.05

4 Convert the following fraction into decimals

a $\frac{3}{5}$

b $\frac{10}{4}$

c $24\frac{54}{1000}$

d $\frac{12}{25}$

e $\frac{8}{25}$

f $1\frac{3}{25}$

5 Addition of decimals

a 4.56 + 32.075 + 256.6245 + 15.0358

b 462.492 + 725.526 + 309.345 + 626.602

6 Subtract the following decimals

a 612.5200 – 9.6479

b 573.9246 – 215.6000

7 Add and subtract the following

a 56.725 + 48.258 – 32.564

b 16.45 + 124.56 + 62.7 - 3.243

8 Multiply the following

a By 10,100,1000

i 3.754 x 10

ii 8.964 x 100

iii 2.3786 x 1000

iv 0.005 x 1000

b By whole numbers

i 8.4 x 7

ii 56.72 x 8

c By another decimal figure (use calculator)

i 15.64 x 7.68

ii 2.642 x 1.562

9 Divide the following

a $\frac{62.5}{25}$

b $\frac{64.56}{10}$

c $\frac{0.42}{100}$

d $\frac{48.356}{1000}$

10 Division

a $\frac{16.8}{1.2}$

b $\frac{1.54}{1.1}$

11 Change the fraction into a decimal

i $1\frac{5}{8}$

ii $\frac{12}{25}$

12 Find the value

20.5 x 40 ÷ 10.25 + 18.50

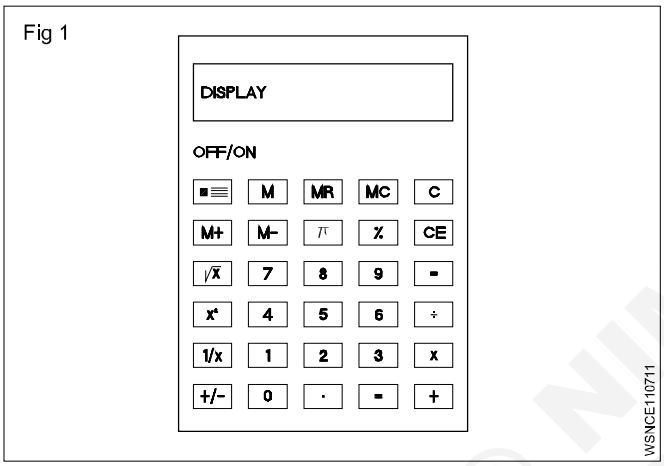
Unit, Fractions - Solving problems by using calculator

A pocket calculator allows to spend less time in doing tedious calculations. A simple pocket calculator enables to do the arithmetical calculations of addition, subtraction, multiplication and division, while a scientific type of calculator can be used for scientific and technical calculations also.

No special training is required to use a calculator. But it is suggested that a careful study of the operation manual of the type of the calculator is essential to become familiar with its capabilities. A calculator does not think and do. It is left to the operator to understand the problem, interpret the information and key it into the calculator correctly.

Constructional Details (Fig 1)

Fig 1



The key board is divided into five clear and easily recognizable areas and the display.

- **Data entry keys**

The entry keys are from [0] .....to [9]

and a key for the decimal point [.] .

- **Clearing keys**

These keys have the letter 'C'

[C] [CLR] Clear totally

[CE] Clear entry only

[CM] , [MC] Clear memory

[+] Addition key

[-] Subtraction key

[x] Multiplication key

[÷] Division key

[=] Equals key to display the result

- **Function keys**

[π] Pi key

[$\sqrt{x}$] Square root key

[%] Percentage key

[+/-] Sign change key

[x^2] Square key

[$\frac{1}{X}$] Reciprocal key

- **Memory keys**

[M] Store the display number

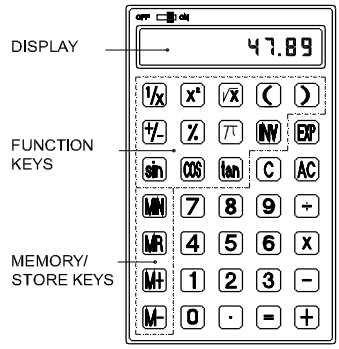
[M+] The displayed value is added to the memory

[M-] The displayed value is subtracted from the memory

[MR] [RCL] The stored value is recalled on to the display

Further functional keys included in Scientific calculators are as shown in Fig 2.

Fig 2



Sin Cos Tan ( ) For trigonometric functions and for brackets

Exp Exponent key

INV Some of the keys have coloured lettering above or below them. To use a function in coloured lettering, press INV key. INV will appear on the display. Then press the key that the coloured lettering identifies. INV will disappear from the display.

log , INV 10^x to obtain the logarithm of the displayed number and the antilogarithm of the displayed value.

INV R–P to convert displayed rectangular coordinates into polar coordinates.

INV P–R to convert displayed polar coordinates into rectangular coordinates.

- **The display**

The display shows the input data, interim results and answers to the calculations.

The arrangement of the areas can differ from one make to another. Keying in of the numbers is done via. an internationally agreed upon set of ten keys in the order that the numbers are written.

Rules and Examples:

- **Addition:** Example 18.2 + 5.7

Sequence	Input	Display
Input of the 1st term of the sum	1 8 . 2	18.2
Press + key	+	18.2
Input 2nd term of the sum. the first term goes into the register	5 . 7	5.7
Press the = key	=	23.9

- **Subtraction:** Example 128.8 - 92.9

Sequence	Input	Display
Enter the subtrahend	1 2 8 . 8	128.8
Press - key	-	128.8
Enter the minuend. The subtrahend goes into the register	9 2 . 9	92.9
Press the = key	=	35.9

- **Multiplication:** Example 0.47 x 2.47

Sequence	Input	Display
Enter multiplicand	. 4 7	0.47
Press x key	x	0.47
Enter multiplier, multiplicand goes to register	2 . 4 7	2.47
Press = key	=	1.1609

- **Division:** Example 18.5/2.5

Sequence	Input	Display
Enter the dividend	1 8 . 5	18.5
Press ÷ Key	÷	18.5
Enter the divisor goes to the register	2 . 5	2.5
Press = key	=	7.4

- **Multiplication & Division:**
  Example : 2.5 x 7.2 / 4.8 x 1.25

Sequence	Input	Display
Enter 2.5	2 . 5	2.5
Press x key	x	2.5
Enter 7.2	7 . 2	7.2
Press ÷ key	÷	18
Enter Open bracket	(	
Enter 4.8	4 . 8	4.8
Press x key	x	4.8
Enter 1.25	1 . 2 5	1.25
Enter Close bracket	)	6
Press = key	=	3.0

- Store in memory Example (2+6) (4+3)

Sequence	Input	Display
Workout for the first bracket	2	2
	+	2
	6	6
	=	8
Store the first result in	STO , M	8
x	or M+	
Workout for the 2nd bracket	4	4
	+	4
	3	3
	=	7
Press x key	x	7
Recall memory	RCL or MR	8
Press = key	=	56

- **Percentage:** Example 12% of 1500

Sequence	Input	Display
Enter 1500	1 5 0 0	1500
Press x key	x	1500
Enter 12	1 2	12
Press INV %	INV %	12
Press = key	=	180

- **Square root:** Example $\sqrt{2+\sqrt{3\times5}}$

Sequence	Input	Display
Enter 2	2	2
Press $\sqrt{a}$ key	$\sqrt{a}$	1.414
Press + key	+	1.414
Press bracket key	(	1.414
Enter 3	3	3
Press $\sqrt{a}$ key	$\sqrt{a}$	1.732
Press x key	x	1.732
Enter 5	5	5
Press $\sqrt{a}$ key	$\sqrt{a}$	2.236
Press bracket close key	)	3.873
Press = key	=	5.2871969
2 √ + (3 √ x 5 √) =		5.2871969

$\sqrt{2+\sqrt{3\times5}}$ = 5.287

- **Common logarithm:** Example log 1.23

Sequence	Input	Display
1 . 2 3 log	=	0.0899051

- **Power:** Example $123 + 30^2$

Sequence	Input	Display
1 2 3 + 3 0 INV X^2	=	1023

- **Before starting the calculations be sure to press the 'ON' key and confirm that '0' is shown on the display.**
- **Do not touch the inside portion of the calculator. Avoid hard knocks and unduly hard pressing of the keys.**
- **Maintain and use the calculator in between the two extreme temperatures of 0° and 40° C.**

- **Never use volatile fluids such as lacquer, thinner, benzine while cleaning the unit.**
- **Take special care not to damage the unit by bending or dropping.**
- **Do not carry the calculator in your hip pocket.**

Assignment

1 Using calculator solve the following

a 625 + 3467 + 20 + 341 + 6278 = _____

b 367.4 + 805 + 0.7 + 7.86 + 13.49 = _____

c 0.043 + 1.065 + 13.0 + 34.76 + 42.1 = _____

d 47160 + 1368.4 + 0.1 + 1.6901 + 134.267 = _____

2 Using calculator simplify the following

a 24367 - 4385 = _____

b 9.643 - 0.7983 = _____

c 4382.01 - 381.3401 = _____

d 693.42 - 0.0254 = _____

3 Using calculator find the values of the following

a 23 x 87 = _____

b 1376 x 0.81 = _____

c 678 x 243 = _____

d 0.75 x 0.24 = _____

4 Using calculator solve the following

a 22434 ÷ 3 = _____

b 4131 ÷ 243 = _____

c 469890 ÷ 230 = _____

d 3.026 ÷ 0.89 = _____

5 Solve the following

a $\frac{1170 \times 537.5}{13 \times 215}$ = _____

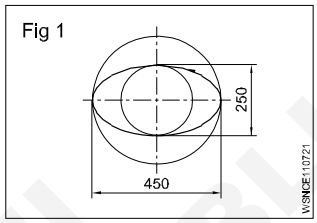
b $\frac{28.2 \times 18 \times 3500}{1000 \times 3 \times 0.8}$ = _____

6 Solve the following

a $\frac{(634 + 128) \times (384 - 0.52)}{8 \times 0.3}$ = _____

b $\frac{(389 - 12.2) \times (842 - 0.05 - 2.6)}{(3.89 - 0.021) \times (28.1 + 17.04)}$ = _____

7



2a = 450 mm(major axis)

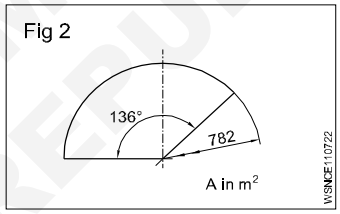
2b = 250mm(minor axis)

Perimeter of the ellipse

A = _____metre^2

Hint A = π x a x b unit^2

8



ø = 782 mm

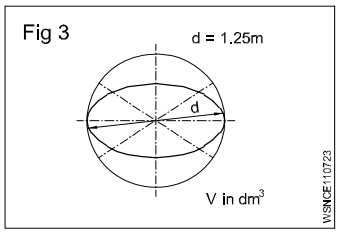
α = 136°

Area of the sector

A = _____

Hint $A = \frac{\pi \times d^2}{4} \times \frac{\alpha}{360°}$

9



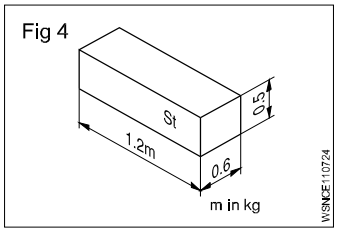
d = 1.25 metre

V = _____ dm^3

Volume of sphere

Hint $V = \frac{4}{3}\pi r^3$

10



L = 1.2 metres

B = 0.6 metre

H = 0.5 metre

'ρ' (rho) density of steel = 7.85 kg/dm^3'

m = _____ kg

(mass 'm = V x ρ)

Square root, Ratio and Proportions, Percentage - Square and square root

a basic number
2 exponent

$\sqrt{\ }$ radial sign indicating the square root.

$\sqrt{a^2}$ square root of 'a' square

a^2 radicand

Square number

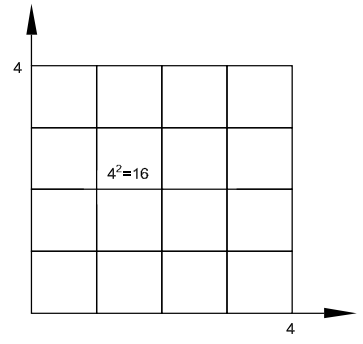
The square of a number is the number multiplied by itself.

Basic number x basic number = Square number

$a \times a = a^2$

$4 \times 4 = 4^2 = 16$

Fig 1



Splitting up

A square area can be split up into sub-areas. The largest square of 36 is made up of a large square 16, a small square 4 and two rectangles 8 each.

Large square 4 x 4 = 16 a^2

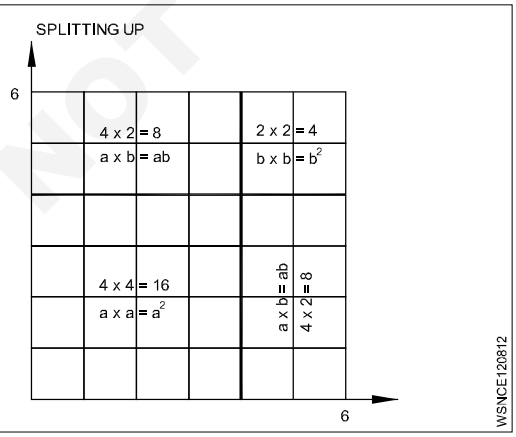
Two rectangles 2 x 4 x 2 = 16 2ab

Small square 2 x 2 = 4 b^2

Sum of sub-areas = 36 = $a^2 + 2ab + b^2$

$\sqrt{36} = \sqrt{a^2 + 2ab + b^2}$

Fig 2

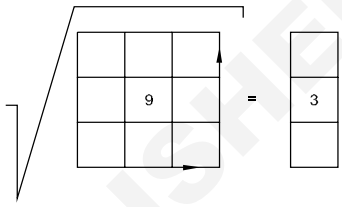


Result: In order to find the square root, we split up the square numbers.

Extracting the square root procedure

- Starting from the decimal point form groups of two figures towards right and left. Indicate by a prime symbol. $\sqrt{4624.00}$
- Find the root of the first group, calculate the difference, bring down the next group.
- Multiply the root by 2 and divide the partial radicand.
- Enter the number thus calculated in the divisor for the multiplication.

Fig 3



If there is a remainder, repeat the procedure.

```
      68
   ------
6  | 46,24
     36
   ------
128| 1024
     1024
   ------
        0
```

$\sqrt{4624} = 68$

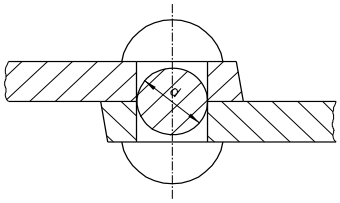
Basic number x basic number = Square

$\sqrt{\text{Square number}}$ = basic number

Example

The cross-section of a rivet is 3.46 cm^2. Calculate the diameter of the hole.

Fig 4



Rivet cross-section is the hole cross-section.

To find 'd',

Given that Area = 3.46 cm^2

Area = 0.785 x d^2 (formula)

3.46 cm^2 = d^2 x 0.785

$$d^2 = \frac{3.46\ cm^2}{0.785}$$

$$d = \sqrt{\frac{3.46}{0.785}}\ cm$$

$d = 2.1\,cm\ (or)\ 21mm$

Square root, Ratio and Proportions, Percentage - Simple problems using calculator

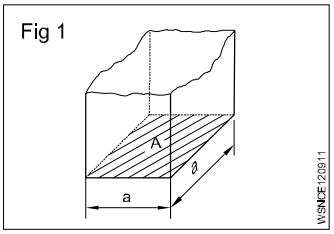
1 a $\sqrt{2916}$ = _____.

b $\sqrt{45796}$ = _____.

c $\sqrt{8.2944}$ = _____.

d $\sqrt{63.845}$ = _____.

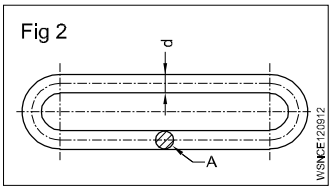
2



A = 2025 mm²

a = _____mm

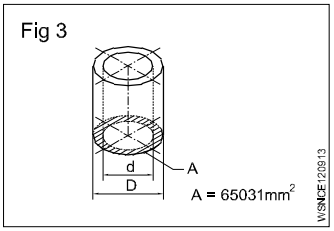
3



A = 176.715 mm²

d = _____mm

4

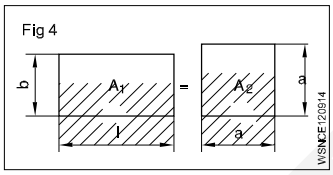


A = 65031 mm²

d = 140 mm

D = _____mm

5



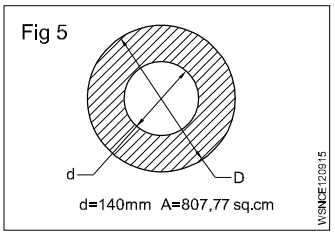
l = 58 cm

b = 45 cm

$A_1 = A_2$

a = _____cm

6

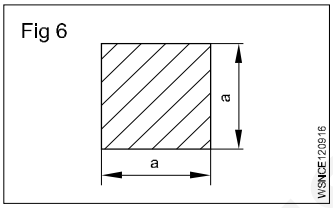


A = 807.77 cm²

d = 140 mm

D = _____mm

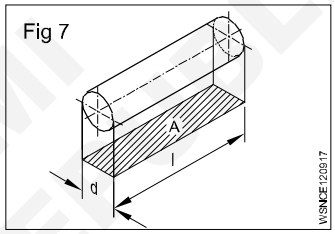
7



a x a = 543169 mm²

a = _____mm

8

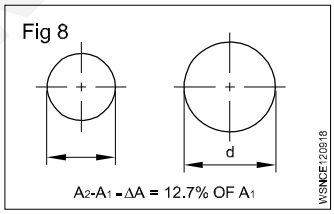


d : l = 1:1.5

A = 73.5 mm²

d = _____mm

9



increase in area

A = 12.7%

A = 360 mm²

d = _____mm

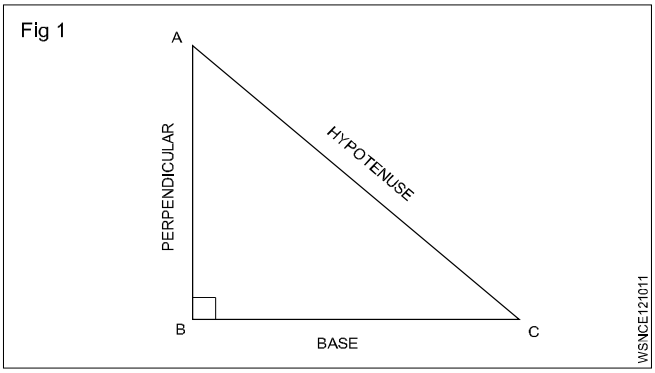
(d = diameter after the increase in area)

Square root, Ratio and Proportions, Percentage - Applications of pythagoras theorem and related problems

Applications of Pythagoras Theorem

Some of the applications of the Pythagoras theorem are; (Fig 1)

Fig 1



1 The Pythagoras theorem is commonly used to find the lengths of sides of a right-angled triangle.

2 It is used to find the length of the diagonal of a square.

3 Pythagoras theorem is used in trigonometry to find the trigonometric ratios like sin, cos, tan, cosec, sec and cot.

4 Pythagoras theorem is used in security cameras for face recognition.

5 Architects use the technique of the Pythagoras theorem for engineering and construction fields.

6 The Pythagoras theorem is applied in surveying the mountains.

7 It is also used in navigation to find the shortest route.

8 By using the Pythagoras theorem, we can derive the formula for base, perpendicular and hypotenuse.

9 Painters use ladders to paint on high buildings with the help of the Pythagoras theorem.

10 Pythagoras theorem is used to calculate the steepness of slopes of hills or mountains.

11 The converse of the Pythagoras theorem is used to check whether a triangle is a right triangle or not.

Application of pythagoras theorem in real life

Pythagoras theorem states that

"In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides".

1 The sides of this triangle have been named Perpendicular, Base and Hypotenuse.

2 The hypotenuse is the longest side, as it is opposite to the angle 90°.

3 The sides of a right triangle (say AB, BC and CA) which have positive integer values, when squared, are put into an equation, also called a Pythagorean triplet.

4 To calculate the length of staircase required to reach a window

5 To find the length of the longest item can be kept in your room.

6 To find the steepness of the hills or mountains.

7 To find the original height of a tree broken due to heavy rain and lying on itself

8 To determine heights and measurements in the construction sites.

Examples

1 What is the side AC if AB = 15 cm, BC = 25 cm.

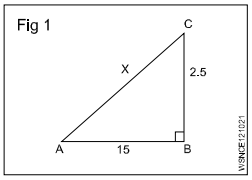
$AC^2 = AB^2 + BC^2$

$= 15^2 + 25^2$

$= 225 + 625 = 850$

$AC = \sqrt{850} = 29.155$ cm

Fig 1



2 What is the side BC if AB = 10 cm, AC = 30 cm.

$AC^2 = AB^2 + BC^2$

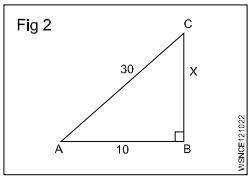
$30^2 = 10^2 + BC^2$

$900 = 100 + BC^2$

$BC^2 = 900 - 100 = 800$

$BC = 28.284$ cm

Fig 2



3 What is the side AB if BC = 20 cm, AC = 35 cm.

$AC^2 = AB^2 + BC^2$

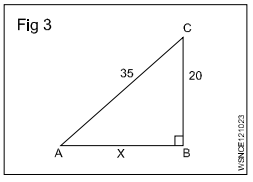
$35^2 = AB^2 + 20^2$

$1225 = AB + 400$

$AB^2 = 1225 - 400 = 825$

$AB = 28.72$ cm

Fig 3



4 What is the value of side BC if AB = 8 cm, AC = 24 cm.

$AC^2 = AB^2 + BC^2$

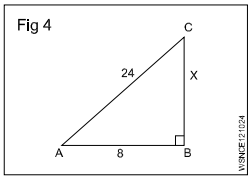
$24^2 = 8^2 + BC^2$

$576 = 64 + BC^2$

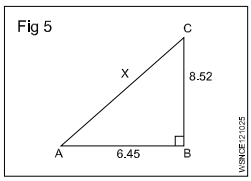
$BC^2 = 576 - 64 = 512$

$BC = \sqrt{572} = 22.63$ cm

Fig 4



5 What is the value side AC if AB = 6.45 cm, BC = 8.52 cm.



$AC^2 = AB^2 + BC^2$

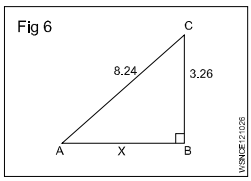
$AC^2 = 6.45^2 + 8.52^2$

$AC^2 = 41.60 + 72.59$

$= 114.19$

$AC = \sqrt{114.19} = 10.69$ cm

6 What is the value of side AB if BC = 3.26 cm, AC = 8.24 cm.



$AC^2 = AB^2 + BC^2$

$8.24^2 = AB^2 + 3.26^2$

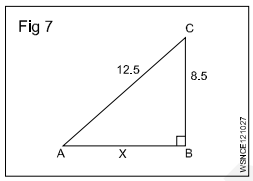
$67.9 = AB + 10.63$

$AB^2 = 67.9 - 10.63$

$= 57.27$

$AB = \sqrt{57.27} = 7.57$ cm

7 What is the value of side AB if AC = 12.5 cm, BC = 8.5 cm.



$AC^2 = AB^2 + BC^2$

$12.5^2 = AB^2 + 8.5^2$

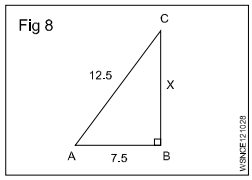
$156.25 = AB + 72.25$

$AB^2 = 156.25 - 72.25$

$= 84$

$AB = \sqrt{84} = 9.17$ cm

8 A ladder of 12.5 metre long is placed with upper end against a wall. The lower end being 7.5 metres from the wall. What height is the upper end above the ground.



$AC^2 = AB^2 + BC^2$

$BC^2 = AC^2 - AB^2$

$BC^2 = x^2$

$AC^2 = AB^2 + BC^2$

$12.5^2 = x^2 + 7.5^2$

$x^2 = (12.5)^2 - (7.5)^2$

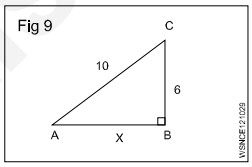
$= (12.5 + 7.5)(12.5 - 7.5)^2$

$= 20 \times 5$

$= \sqrt{100} = 10$

$x = 10$ m

9 What is the value of AB.



$AC^2 = AB^2 + BC^2$

$AB^2 = AC^2 - BC^2$

$AB^2 = x^2$

$AC^2 = AB^2 + BC^2$

$10^2 = x^2 + 6^2$

$x^2 = 10^2 - 6^2$

$= 100 - 36 = 64$

$x = \sqrt{64}$

$= 8$

$AB = 8$

Assignment

1 What is the value of side AB, in a right angled triangle of side AC = 10 cm and BC = 5 cm.

2 What is the value of side AC, in a right angled triangle of side AB = 6.5 cm and BC = 4.5 cm.

3 What is the value of side BC, in a right angled triangle of side AC = 14.5 cm and AB = 10.5 cm.

4 What is the value of side AC, in a right angled triangle of side AB = 7 cm and BC = 5 cm.

5 What is the value of side BC, in a right angled triangle of side AC = 13.25 cm and AB = 8.75 cm.

Square root, Ratio and Proportions, Percentage - Ratio and proportion

Ratio

Introduction

It is the relation between two quantities of the same kind and is expressed as a fraction.

Expression

a, b two quantities of the same kind. $\frac{a}{b}$ or a:b or a $\div$ b or a in b is the ratio.

Ratio is always reduced to the lowest terms.

Example

$$7:14 = \frac{7}{14} = \frac{1}{2} = 1:2$$

Proportion

It is the equality between the ratios, a : b is a ratio and c: d is another ratio. Both ratios are equal. Then

a :b :: c : d or $\frac{a}{b} = \frac{c}{d}$

Example

250 : 2000 :: 1 : 8

Proportion fundamentals

If $\frac{a}{b} = \frac{c}{d}$ then

- ad = bc
- $\frac{a}{c} = \frac{b}{d}$
- $\frac{b}{a} = \frac{d}{c}$
- $\frac{a+b}{b} = \frac{c+d}{c}$ and $\frac{a+b}{a} = \frac{c+d}{c}$
- $\frac{a-b}{b} = \frac{c-d}{d}$
- $\frac{a+b}{b+d} = \frac{a}{c} = \frac{c}{d}$

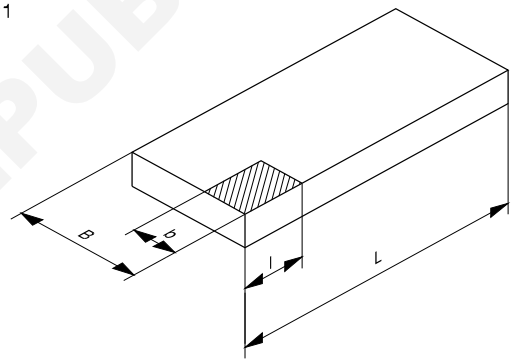
  3:4::6:8 or $\frac{3}{4} = \frac{6}{8}$
- 3 x 8 = 6 x 4
- $\frac{3}{6} = \frac{4}{8}$
- $\frac{4}{3} = \frac{8}{6}$
- $\frac{3+4}{4} = \frac{6+8}{8}$
- $\frac{3-4}{4} = \frac{6-8}{8}$
- $\frac{3+6}{4+8} = \frac{9}{12} = \frac{3}{4}$

Ratio - relation of two quantities of the same kind.
Proportion - equality between two ratios.

Example

- A steel plate of 800 x 1400 mm is to be drawn to a scale of 1:20. What will be the lengths in the Fig 1.

Fig 1



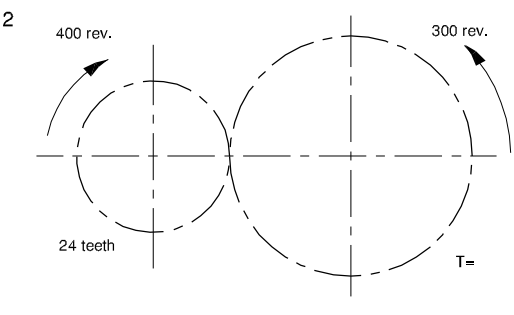
The reduction ratio is $\frac{1}{20}$.

B is reduced from 800 to 800 x $\frac{1}{20}$ = 40 mm.

L is reduced from 1400 x $\frac{1}{20}$ = 70 mm.

- Find the number of teeth of the larger gear in the gear transmission shown in the Fig 2.

Fig 2



Speed ratio = 400 : 300

Teeth ratio = 24:T

$$\frac{400}{300}=\frac{T}{24}$$

$$\therefore T=\frac{24\times400}{300}=32\,\text{Teeth}$$

Find the ratio of A:B:C

If A:B= 2:3 and B:C=4:5

A:B = 2:3

B:C = 4:5

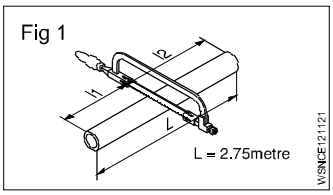
A:B = 8 :12 (Ratio 2:3 multiply by 4)

B:C = 12:15 (Ratio 4:5 multiply by 3)

∴ A:B:C = 8:12:15

Assignment

1



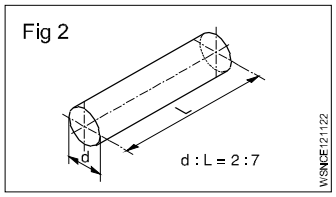
$l_1 : l_2$ = 2:3

L = 2.75 metres

l_1 = _____ metres

l_2 = _____ metres

2

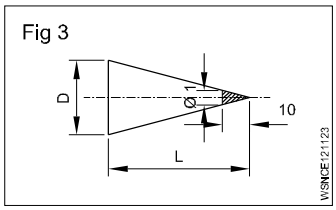


d: L of shaft = 2:7

d = 40 mm

L = _____ mm

3

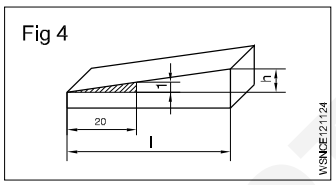


D : L=1:10

L=150mm

D=_____mm

4

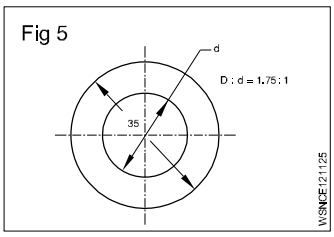


$$\frac{\Delta h}{l}=\frac{1}{20}$$

l = 140 mm

Δh = _____ mm

5

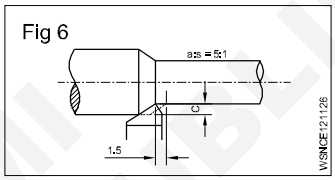


D : d = 1.75: 1

D = 35 mm

d = _____ mm

6



a:s = 5:1

s = 1.5mm

a =_____ mm

7 A:B=9:12

B:C=8:10

Then A:B:C=_____

8 A:B=5:6

B:C=3:4

Then A:B:C= _____

9 A:55=9:11

A = _____

10 15:9.3=40:x

x = _____

Square root, Ratio and Proportions, Percentage - Ratio and Proportions - Direct and indirect proportions

Proportion

Description

It is the equality between the ratios, a:b is a ratio and c:d is another ratio. Both ratios are equal. Then

a : b::c : d or e.g. 250 : 2000::1 : 8

Rule of three

A three step calculation

statement

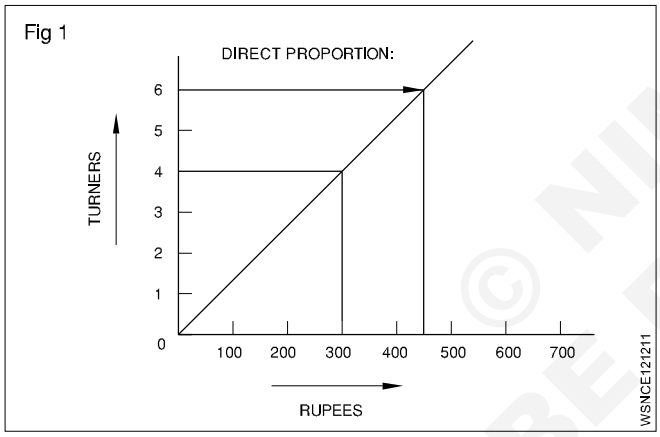
single

multiple.

Direct proportion

The more in one the more in the other - An increase in one denomination produces an increase in the other. (Fig 1)

Fig 1



Examples

1 4 turners earn 300 Rupees. How much will 6 Turners earn?

Statement

4 turners = 300 Rupees

Single

1 Turner = 75 Rupees

Multiple

6 Turners = 6 x 75 = 450 Rupees

2 One vehicle consumes 30 litres of petrol per day how much petrol is used by 6 Vehicles operating under similar condition.

One vehicle uses petrol = 30 litres per day.

Then six vehicles will use = 6 Times as much

= 6 x 30 = 180 litres/day.

3 4 vehicles consumes 120 gallons of petrol per day how much petrol will be used by 12 vehicles operating under the same condition.

4 vehicles use 120 gallons per day

1 Vehicle will use $\frac{120}{4}$ = 30 gallons/day

12 vehicles will use 12 x 30 = 360 gallons/day

Both examples are called simple proportion because only two quantities were used and the day is common for both ratios.

4 If 2 litres of petrol costs Rs 60. Find the cost of 50 litres.

Quantity of Petrol	Cost of Petrol
2 litres	Rs.60
50 litres	x

1 litre petrol = $\frac{60}{2}$ = Rs.30

50 litres petrol = 30 x 50 = Rs 1500

5 A 150mm dia gear meshes with 50mm dia gear. If the larger gear has 30 teeth. How many teeth will have the smaller gear have?

Gear dia	No. of Teeth
150 mm	30
50 mm	x

$$x = \frac{50}{150} \times 30 = 10 \text{ teeth.}$$

6 A mechanic assembles 7 machines in 2½ days. How long will it take time to assemble 70 machines at the same rate.

Machines	Days
7	2½
70	x

$$x = \frac{70 \times 2.5}{7} = 25 \text{ days}$$

Assemble for 70 machines will take 25 days.

7 A roll of wire weighs 1.24 kg from this roll a piece of 3.7cm long is cut and it is found to weigh 2.93 gm. What is the length of the wire in the roll?

Weight of wire	Length of wire
2.93 gm	3.7 cm
1.24 kg (1240 gm)	x

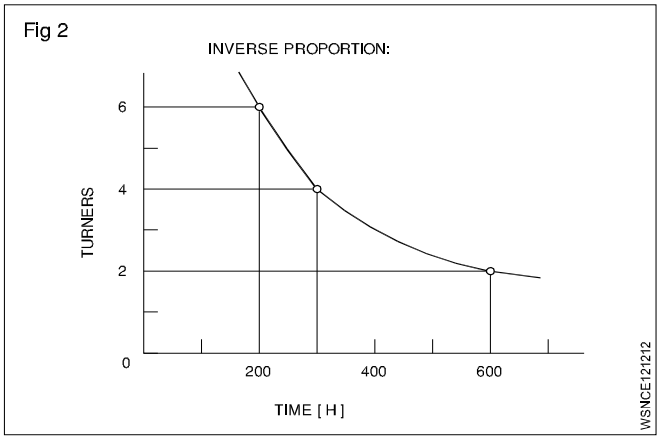
$$x = \frac{1240}{2.93} \times 3.7 = 1566 \text{ cm}$$

Length of wire = 1566 cm.

Indirect or inverse proportion

The more in one the lesser other - Increase in one quantity will produce a decrease in the other. (Fig 2)

Fig 2



Example

1 4 turners finish a job in 300 hours. How much time will 6 turners take to do the same job?

Solution procedure in three steps:

Statement 4 turners taken = 300 hours

The time will reduce if 6 turners to do the same job. Therefore this is inverse proportion.

Multiple fraction $\frac{4 \text{ Turners}}{6 \text{ Turners}} \times 300 \text{ hours}$

6 Turners = 200 hours

Result - The more the less.

2 8 workman take 6 days to complete a job. How many days it will take for 4 workman to complete the same job?

Workman	Days
8	6
4	x

$$x = \frac{8}{4} \times 6 = 12 \text{ days}$$

4 workers complete the work = 12 days.

3 5 men working on a job finished it in 32 days. Find out in how many days 8 men will finish the same job?

Men	Days
5	32
8	x

$$x = \frac{5 \times 32}{8} = 4 \times 5 = 20 \text{ days}$$

8 men will complete the job = 20 days.

4 An engine running at 150 rpm drives a shaft by pulley diameter is 55cm and that of the driven shaft pulley is 33 cm. Find the speed of the shaft?

Dia of pulley	Rpm of shaft
55 cm	150
33 cm	x

$$x = \frac{55 \times 150}{33} = 250 \text{ rpm.}$$

Speed of the 33cm diameter will run 250 rpm.

5 A pulley of 80 cm diameter is rotating at 100 rpm and drives another pulley of 40 cm diameter. Find the rpm of driven pulley. If slip is 2.5% find the rpm?

Dia of pulley	Rpm of pulley
80 cm	100
40 cm	x

40 cm diameter = 200 rpm.

Slip is 2.5% = 195 rpm.

Problems involving both

Example

2 turners need 3 days to produce 20 pieces. How long will it take for 6 turners to produce 30 such pieces?

Statement

2 turners, 20 pieces = 3 days

6 turners, 30 pieces = how many days.

First step (Fig 3)

Statement 2 turners for 20 pieces = 3 days

1 turner for 20 pieces = 3 x 2 = 6 days

Multiple 6 turners for 20 pieces = $\frac{6}{6}$ = 1 day

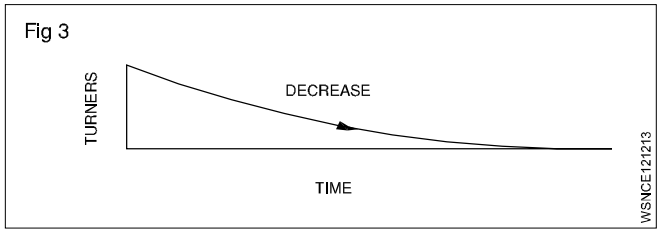
Statement 6 turners for 20 pieces = 1 day

Single 6 turners for 1 piece = $\frac{1}{20}$ days

Multiple 6 turners for 30 pieces = $\frac{1}{20} \times 30$ = 1.5 days

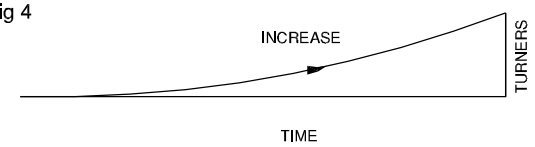
Inverse proportion - More the less.

Fig 3



Second step (Fig 4)

Fig 4



Direct proportion - More the more.

Solve the probLem by first writing the statement and proceed to single and then to the multiple according to the type of proportion that is involved.

Introduction

Proportional fundamentals, as applicable to motor vehicle calculations are discussed below.

Simple Proportion

- Proportion

This is an equality between two ratios

Compound and Inverse proportions

- Compound proportions

Example

5 Fitter take 21 days to complete overhauling of 6 vehicles how long 7 Fitters will take to over haul 8 vehicles (Assume time of overhauling each vehicle is constant)

In this both direct and indirect proportions are used.

- 1 Fitter will overhauling 1 vehicle in days (shorter time).
- Quantities (No. of days) are taken in last as that is the answer required in this case.

Fitters	Vehicle	Days
5	6	21
7	8	x

$$\left(\frac{21\times5}{6\times7}\times8\right)=20 \text{ days}$$

Ans: 7 Fitters will overhaul 8 vehicles in 20 days.

Inverse proportion

Some times proportions are taken inversely.

Examples

- If one water pump fills the fuel tank in 12 minutes, two pumps will take half the time taken.

The time should not be doubled.

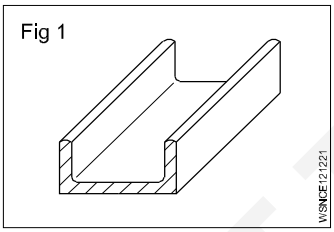
- 2 pumps will take 30 minutes to fill up a tank how long will 6 similar pumps take this to fill this tank.

Pump	Time
2	30
6	x

Ans: Time taken by 6 pumps = $\frac{30\times2}{6}=10$ minutes

Assignment

1 Fig 1

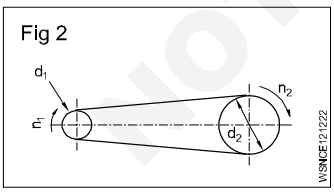


Length = 6.1 metre

Weight = 32 kgf

Weight of 1 metre of the same channel = _____ kgf

2 Fig 2



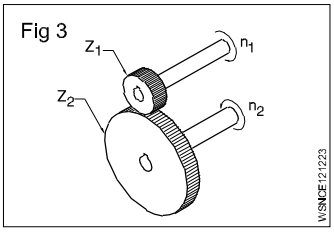
d_1 = 120 mm

d_2 = 720 mm

n_1 = 1200 rpm

n_2 = _____ rpm

3 Fig 3



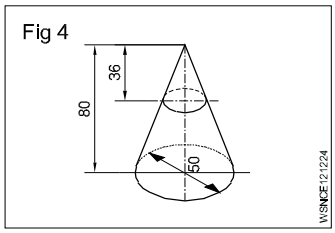
Z_1 = 42 T

n_2 = 96 rpm

n_1 = 224 rpm

Z_2 = _____ T

4 Fig 4



D = 50 mm

H = 80 mm

h = 36 mm

d = _____ mm

5 If a mechanic assembles 8 machines in 3 days, how long he will take to assemble 60 machines.

6 In an auto shop the grinding wheel makes 1000 rpm and the driven pulley is 200 mm dia. If the driving pulley is 150 mm dia. Find out the rpm of the driving pulley.

7 In a gearing of a vehicle the following facts are found.

A 180 mm dia of gear meshes with 60mm dia gear. If the bigger gear makes 60 rpm. What will be the rpm of smaller gear.

8 A vehicular job is completed by 5 mechanics in 4 days. If only 3 mechanics are available, in how many days the work can be completed.

Square root, Ratio and Proportions, Percentage - Percentage

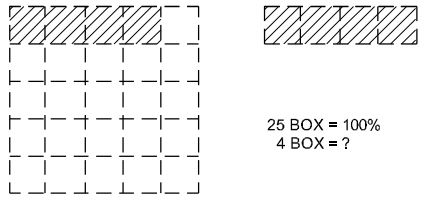
Percentage

Percentage is a kind of fraction whose denominator is always 100. The symbol for percent is %, written after the number. e.g. 16%.

Ex. $\frac{16}{100} = 0.16$

In decimal form, it is 0.16. Percentage calculation also involves rule of three. The statement (the given data), for unit, and then to multiple which is for calculating the answer. (Fig 1)

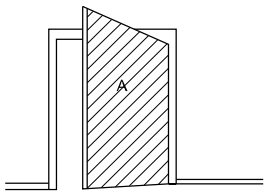
Fig 1



Example

The amount of total raw sheet metal to make a door was 3.6 metre^2 and wastage was 0.18 metre^2. Calculate the % of wastage. (Fig 2)

Fig 2



Solution procedure in three steps.

Statement:

Area of door (A) =3.6 m^2 = 100 %.

Wastage = 0.18 m^2

Single: $\frac{100}{3.6}$ x 1 m^2

Multiple: for 0.18 m^2= $\frac{100}{3.6}$ x 0.18.

Wastage = 5%.

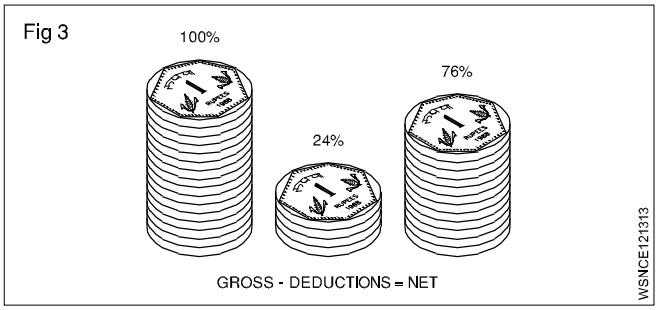
Analyse the given data and proceed to arrive at the answer through the unit.

Example

A fitter receives a take-home salary of 984.50 rupees.

If the deduction amounts to 24%, what is his total salary? (Fig 3)

Fig 3



Total pay 100%

Deduction 24%

Take home salary 76%

If the take home pay is Rs.76, his salary is 100.

For 1% it is $\frac{1}{76}$

For Rs.984.50, it is $\frac{1}{76}$ x 984.50.

For 100% it is $\frac{984.50}{76} X100 = 1295.39$

100% i.e. gross pay = Rs.1295.40.

Example 1

75 litres of oil is taken out from a oil barrel of 200 litres capacity. Find out the percentage taken in this.

Solution

% of oil taken = Oil taken out (litres) / Capacity of Barrel (litres) x 100

$= \frac{75}{200} \text{ x } 100 = 37\frac{1}{2}\%$

Example 2

A spare part is sold with 15%. Profit to a customer, to a price of Rs.15000/-. Find out the following (a) What is the purchase price (b) What is the profit.

Solution: CP = x,

CP = cost price

SP = sale price

SP=CP+15%of CP

$15000 = x + \frac{15x}{100} = \frac{100x + 15x}{100}$

$x = \frac{1500000}{115} = 13043.47$

Profit = SP-CP = 15000-13043.47 = 1956.53

Purchase price = Rs.13,043/,Profit = Rs. 1957

Example 3

Out of 80000 cars, which were tested on road, only 16000 cars had no fault. What is the percentage in this acceptance.

$$= \frac{16000}{80000} \times 100 = \frac{100}{5} = 20\%$$

Example 4

The price of a motor cycle dropped to 92% of original price and now sold at Rs.18000/- What was the original price.

Solution

Present price of Motor cycle Rs.18000

This is the value of 92% of original price

$$\text{Original Price} = 18000 \times \frac{100}{92} = \frac{1800000}{92}$$

= Rs.19565

Example 5

A Motor vehicle uses 100 litres of Petrol per day when travelling at 30 kmph. After top overhauling the consumption falls to 90 litres per day. Calculate percentage of saving.

Solution

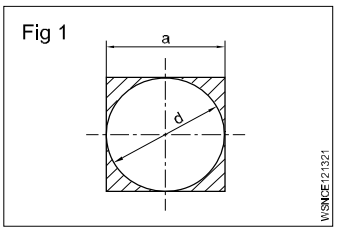
Percentage of saving = Decrease in consumption/Original consumption x 100

$$= (100 - 90)\frac{\text{litres}}{100} \times 100$$

$$= \frac{10}{100} \times 100$$

= 10% Saving in fuel.

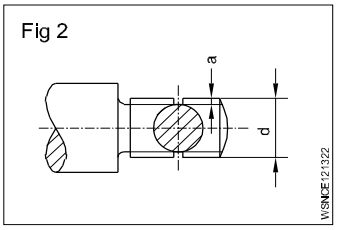
Assignment

1 

a = 400mm (side of square)

d = 400 mm

Wastage = _____ %.

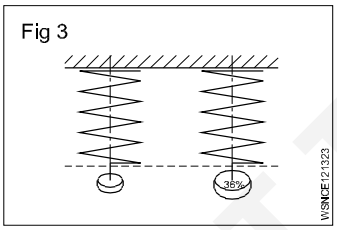
2 

d = 26mm

'a' depth of u/cut = 2.4mm

reduction of area at cross-section

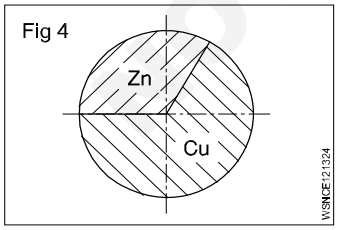
= _____ %

3 

Percentage of increase = 36%

Value of increase = 611.2 N/mm^2

Original tensile strength = _____ N/mm^2.

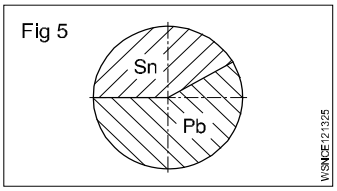
4 

Copper in alloy = 27 kg

Zinc in alloy = 18 kg

% of Copper = _____ %

% of Zinc = _____%.

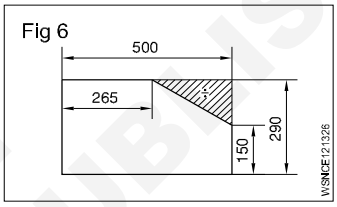
5 

Weight of alloy = 140 Kgf

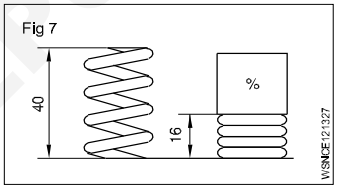
Weight of Sn 40%

Pb = _____ Kgf

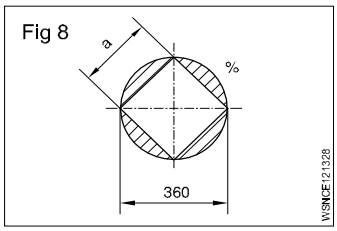
Sn = _____ Kgf.

6 

Shaded portion = _____ %.

7 

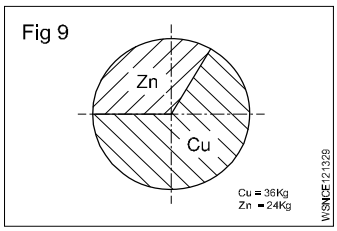
Compression length = _____%.

8 

d = 360 mm

a = 0.707 x d

Wastage = _____ %.

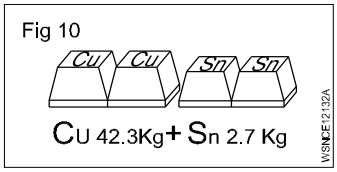
9 

Cu = 36 Kg

Zn = 24 Kg

Cu = _____ %

Zn = _____ %

10 

Cu = 42.3 Kg

Sn = 2.7 Kg

Cu _____ %

Sn = _____ %.

Square root, Ratio and Proportions, Percentage - Changing percentage to decimal and fraction

Conversion of Fraction into Percentage

1 Convert $\frac{1}{2}$ into percentage.

Solution: $\frac{1}{2}$ x 100

= 50%

2 Convert $\frac{1}{11}$ into percentage

Solution: $\frac{1}{11}$ x 100 = $\frac{100}{11}$

= 9.01%

Convert the following fraction into percentage.

1 $\frac{1}{4}$

2 $\frac{1}{5}$

3 $\frac{2}{3}$

4 $\frac{3}{8}$

Conversion of Percentage into Fraction

1 Convert 24% into fraction.

Solution: $\frac{24}{100} = \frac{6}{25}$

2 Convert $33\frac{1}{3}$% into fraction.

Solution: $\frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{100}{3} \times \frac{1}{100}$

$= \frac{1}{3}$

Convert the following percentage into fraction

1 15%

2 $87\frac{1}{2}$%

3 80%

4 12.5%

Conversion of Decimal Fraction into Percentage

1 Convert 0.35 into percentage.

Solution: 0.35 x 100

= 35%

2 Convert 0.375 into percentage.

Solution: 0.375 x 100

= 37.5%

Convert the following Decimal Fraction into Percentage

1 0.2

2 0.004

3 0.875

4 0.052

Conversion of Percentage into Decimal fraction

1 Convert 30% into decimal fraction.

Solution: $\frac{30}{100}$ = 0.3

2 Convert $33\frac{1}{3}$% into decimal fraction.

Solution: $\frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{100}{3} \times \frac{1}{100}$

$= \frac{1}{3}$ = 0.333

Convert the following percentage into decimal fraction

1 15%

2 7%

3 $12\frac{1}{2}$%

4 90%

Material science - Types of metal, types of ferrous and non ferrous metals

Types of metals

The metals is of two types:

1 Ferrous metal 2 Non-ferrous metal

1 Ferrous metals : The metals that contains major part of iron and contain carbon are called ferrous metals such as pig iron, mild steel, nickel etc., they have iron properties such as rusting, magnetisations etc.

2 Non-ferrous metals : The metals that do not contains iron or carbon and do not have the property of iron are called non-ferrous metals such as copper, aluminum etc.

Ferrous and Non ferrous alloys

Alloying metals and ferrous alloys

An alloy is formed by mixing two or more metals together by melting.

For ferrous metals and alloys, iron is the main constituent metal. Depending on the type and percentage of the alloying metal added, the property of the alloy steel will vary.

Metals commonly used for making alloy steels

Nickel (Ni)

This is a hard metal and is resistant to many types of corrosion rust.

It is used in industrial applications like nickel, cadmium batteries, boiler tubes, valves of internal combustion engines, engine spark plugs etc. The melting point of nickel is 1450°C. Nickel can be magnetised. In the manufacture of permanent magnets a special nickel steel alloy is used. Nickel is also used for electroplating. Invar steel contains about 36% nickel. It is tough and corrosion resistant. Precision instruments are made of Invar steel because it has the least coefficient of expansion.

Nickel-steel alloys are available containing nickel from 2% to 50%.

Chromium (Cr)

Chromium, when added to steel, improves the corrosion resistance, toughness and hardenability of steel. Chromium steels are available which may contain chromium up to 30%.

Chromium, nickel, tungsten and molybdenum are alloyed for making automobile components and cutting tools.

Chromium is also used for electroplating components. Cylinder liners are chrome-plated inside so as to have wear resistance properties. Stainless steel contains about 13% chromium. Chromium-nickel steel is used for bearings. Chrome-vanadium steel is used for making hand tools like spanners and wrenches.

Manganese (Mn)

Addition of manganese to steel increases hardness and strength but decreases the cooling rate.

Manganese steel can be used to harden the outer surface for providing a wear resisting surface with a tough core. Manganese steel containing about 14% manganese is used for making agricultural equipment like ploughs and blades.

Silicon (Si)

Addition of silicon for alloying with steel improves resistance to high temperature oxidation.

This also improves elasticity, and resistance against corrosion. Silicon alloyed steels are used in manufacturing springs and certain types of steel, due to its resistance to corrosion. Cast iron contains silicon about 2.5%. It helps in the formation of free graphite which promotes the machinability of cast iron.

Tungsten (W)

The melting temperature of tungsten is 3380° C. This can be drawn into thin wires.

Due to this reason it is used to make filaments of electric lamps.

Tungsten is used as an alloying metal for the production of high speed cutting tools. High speed steel is an alloy of 18% tungsten, 4% chromium and 1% vanadium.

Stellite is an alloy of 30% chromium, 20% tungsten, 1 to 4% carbon and the balance cobalt.

Vanadium (Va)

This improves the toughness of steel. Vanadium steel is used in the manufacture of gears, tools etc. Vanadium helps in providing a fine grain structure in tool steels.

Chrome-vanadium steel contains 0.5% to 1.5% chromium, 0.15% to 0.3% vanadium, 0.13% to 1.10% carbon.

This alloy has high tensile strength, elastic limit and ductility. It is used in the manufacture of springs, gears, shafts and drop forged components.

Vanadium high speed steel contains 0.70% carbon and about 10% vanadium. This is considered as a superior high speed steel.

Cobalt (Co)

The melting point of cobalt is 1495°C. This can retain magnetic properties and wear- resistance at very high temperatures. Cobalt is used in the manufacture of magnets, ball bearings, cutting tools etc. Cobalt high speed steel (sometimes known as super H.S.S.) contains about 5 to 8% cobalt. This has better hardness and wear resistance properties than the 18% tungsten H.S.S.

Molybdenum (Mo)

The melting point of molybdenum is 2620°C. This gives high resistance against softening when heated. Molybdenum high speed steel contains 6% of molybdenum, 6% tungsten, 4% chromium and 2% vanadium. This high speed steel is very tough and has good cutting ability.

Cadmium (cd)

The melting point of cadmium is 320°C. This is used for coating steel components.

Alloying Metals and Non Ferrous Alloys

Non-ferrous Metals And Alloys

Copper and its alloys

Metals without iron are called non-ferrous metals. Eg. Copper, Aluminium, Zinc, Lead and Tin.

Copper

This is extracted from its ores 'MALACHITE' which contains about 55% copper and 'PYRITES' which contains about 32% copper.

Properties

Reddish in colour. Copper is easily distinguishable because of its colour.

The structure when fractured is granular, but when forged or rolled it is fibrous.

It is very malleable and ductile and can be made into sheets or wires.

It is a good conductor of electricity. Copper is extensively used as electrical cables and parts of electrical apparatus which conduct electric current.

Copper is a good conductor of heat and also highly resistant to corrosion. For this reason it is used for boiler fire boxes, water heating apparatus, water pipes and vessels in brewery and chemical plants. Also used for making soldering iron.

The melting temperature of copper is 1083° C.

The tensile strength of copper can be increased by hammering or rolling.

Copper Alloys

Brass

It is an alloy of copper and zinc. For certain types of brass small quantities of tin or lead are added. The colour of brass depends on the percentage of the alloying elements. The colour is yellow or light yellow, or nearly white. It can be easily machined. Brass is also corrosion-resistant.

Brass is widely used for making motor car radiator core and water taps etc. It is also used in gas welding for hard soldering/brazing. The melting point of brass ranges from 880 to 930°C.

Brasses of different composition are made for various applications.

Bronze

Bronze is basically an alloy of copper and tin. Sometimes zinc is also added for achieving certain special properties. Its colour ranges from red to yellow. The melting point of bronze is about 1005°C. It is harder than brass. It can be easily machined with sharp tools. The chip produced is granular. Special bronze alloys are used as brazing rods.

Bronze of different compositions are available for various applications.

Lead and its alloys

Lead is a very commonly used non-ferrous metal and has a variety of industrial applications.

Lead is produced from its ore 'GALENA'. Lead is a heavy metal that is silvery in colour when molten. It is soft and malleable and has good resistance to corrosion. It is a good insulator against nuclear radiation. Lead is resistant to many acids like sulphuric acid and hydrochloric acid.

It is used in car batteries, in the preparation of solders etc. It is also used in the preparation of paints.

Lead Alloys

Babbitt metal

Babbitt metal is an alloy of lead, tin, copper and antimony. It is a soft, anti-friction alloy, often used as bearings.

An alloy of lead and tin is used as 'soft solder'.

Zinc and its alloys

Zinc is a commonly used metal for coating on steel to prevent corrosion. Examples are steel buckets, galvanized roofing sheets, etc.

Zinc is obtained from the ore-calamine or blende.

Its melting point is 420° C.

It is brittle and softens on heating; it is also corrosion-resistant. Due to this reason it is used for battery containers and is coated on roofing sheets etc.

Galvanized iron sheets are coated with zinc.

Tin and tin alloys

Tin

Tin is produced from cassiterite or tinstone. It is silvery white in appearance, and the melting point is 231° C. It is soft and highly corrosion-resistant.

It is mainly used as a coating on steel sheets for the production of food containers. It is also used with other metals, to form alloys.

Example: Tin with copper to form bronze. Tin with lead to form solder. Tin with copper, lead and antimony to form Babbitt metal.

Aluminium

Aluminium is a non-ferrous metal which is extracted from 'BAUXITE'. Aluminium is white or whitish grey in colour. It has a melting point of 660° C. Aluminium has high electrical and thermal conductivity. It is soft and ductile, and has low tensile strength. Aluminium is very widely used in aircraft industry and fabrication work because of its lightness. Its application in the electrical industry is also on the increase. It is also very much in use in household heating appliances.

Material science - Physical and mechanical properties of metals

Metal:

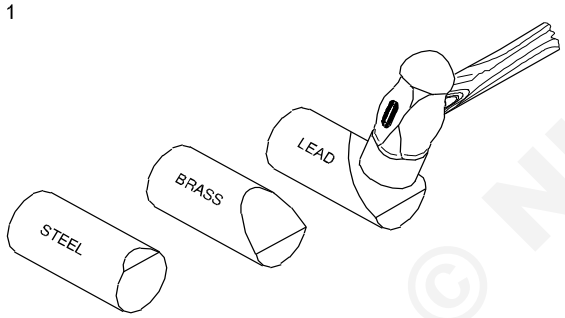
Metal is a mineral used in all types of engineering works such as machineries, bridges, aero planes etc., so we must have basic knowledge about the metals.

Understanding the physical and mechanical properties of metals has become increasingly important for a machinist since he has to make various components to meet the designed service requirements against factors, such as the raise of temperature, tensile, compressive and impact loads etc. A knowledge of different properties of materials will help him to do his job successfully. If proper material/ metal is not used it may cause fracture or other forms of failures, and endanger the life of the component when it is put into function.

Fig 1 shows the way in which the metals get deformed when acted upon by the same load.

Note the difference in the amount of deformation.

Fig 1



Physical properties of metals

- Colour
- Weight/specific gravity
- Structure
- Conductivity
- Magnetic property
- Fusibility

Colour

Different metals have different colours. For example, copper is distinctive red colour. Mild steel is blue/black sheen.

Weight

Metals may be distinguished, based on their weights for given volume. Metals like aluminium lighter weight (Specific gravity 2.7) and metals like lead have a higher weight. (Specific gravity 11.34)

Structure (Figs 2&3)

Generally metals can also be differentiated by their internal structures while seeing the cross-section of the bar through a microscope. Metals like wrought iron and aluminium have a fibrous structure and metals like cast Iron and bronze have a granular structure.

Fig 2

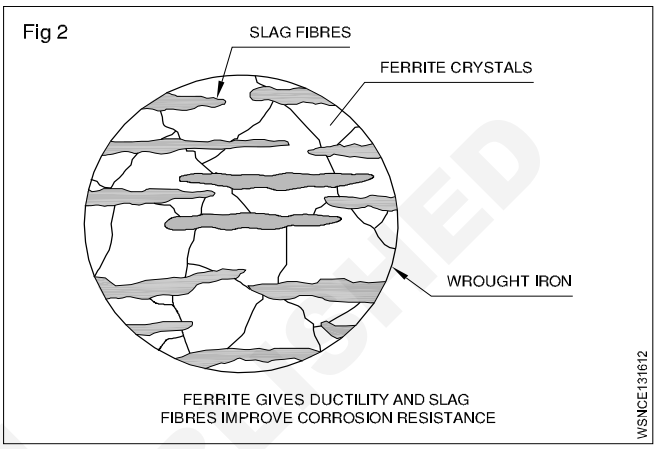
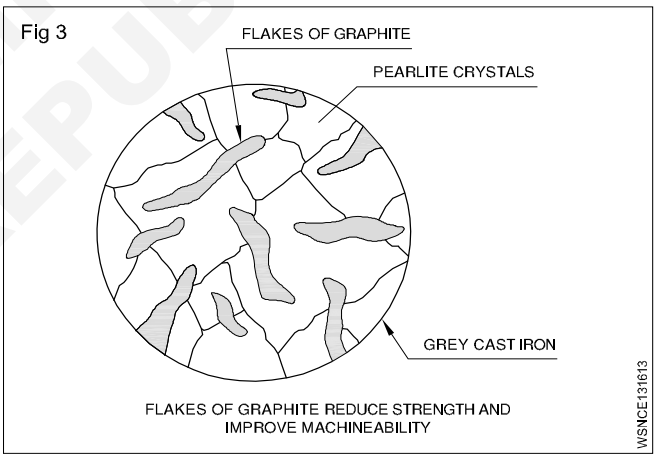


Fig 3



Conductivity (Figs 4&5)

Thermal conductivity and electrical conductivity are the measures of ability of a material to conduct heat and electricity. Conductivity will vary from metal to metal. Copper and aluminium are good conductors of heat and electricity.

Fig 4

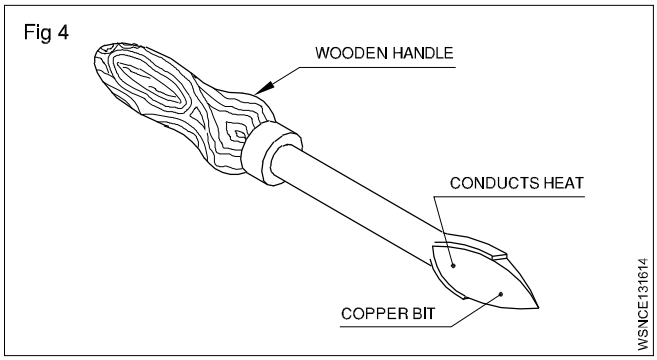
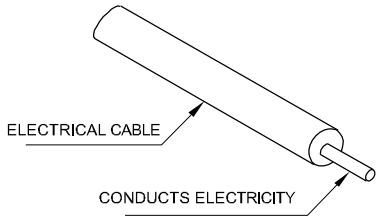


Fig 5



Magnetic property

A metal is said to possess a magnetic property if it is attracted by a magnet.

Almost all ferrous metals, except some types of stainless steel, can be attracted by a magnet, and all non-ferrous metals and their alloys are not attracted by a magnet.

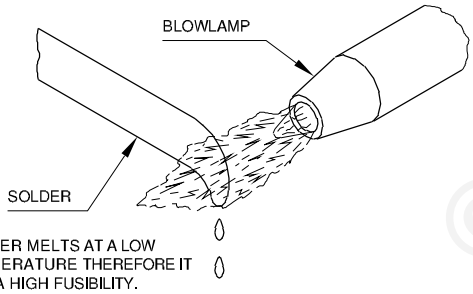
Fusibility (Fig 6)

It is the property possessed by a metal by virtue of which it melts when heat is applied. Many materials are subject to transformation in the shape (i.e) from solid to liquid at different temperatures. Lead has a low melting temperature while steel melts at a high temperature.

Tin melts at 232°C.

Tungsten melts at 3370°C.

Fig 6



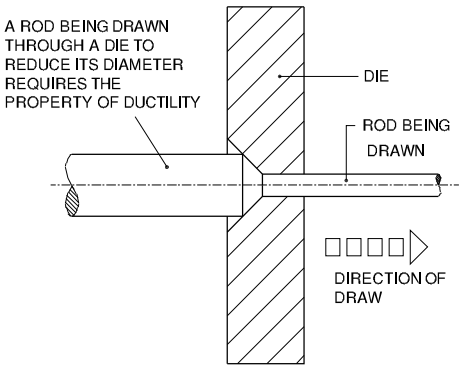
Mechanical properties

- Ductility
- Malleability
- Hardness
- Brittleness
- Toughness
- Tenacity
- Elasticity

Ductility (Fig 7)

A metal is said to be ductile when it can be drawn out into wires under tension without rupture. Wire drawing depends upon the ductility of a metal. A ductile metal must be both strong and plastic. Copper and aluminium are good examples of ductile metals.

Fig 7



Malleability (Figs 8 and 9)

Malleability is the property of a metal by which it can be extended in any direction by hammering, rolling etc. without causing rupture. Lead is an example of a malleable metal.

Fig 8

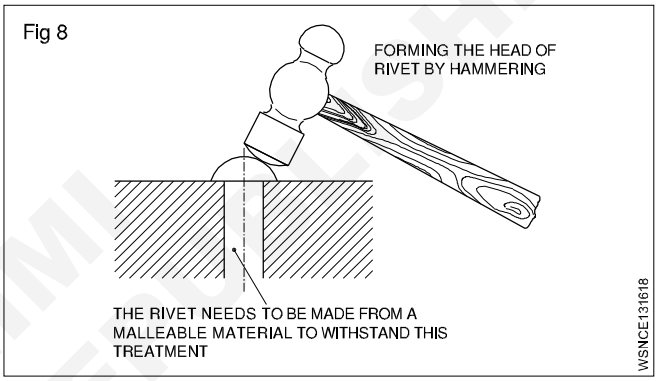
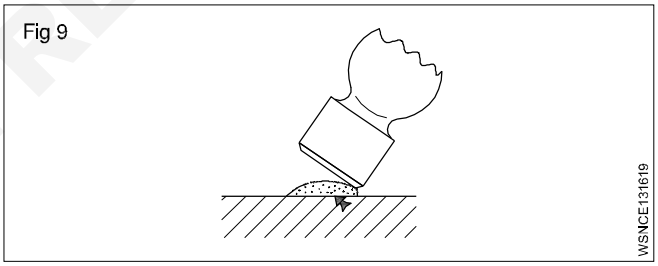


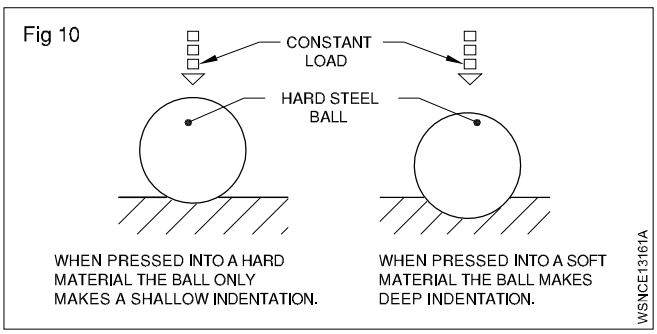
Fig 9



Hardness (Fig 10)

Hardness is a measure of a metal's ability to withstand scratching, wear and abrasion, indentation by harder bodies. The hardness of a metal is tested by marking by a file etc.

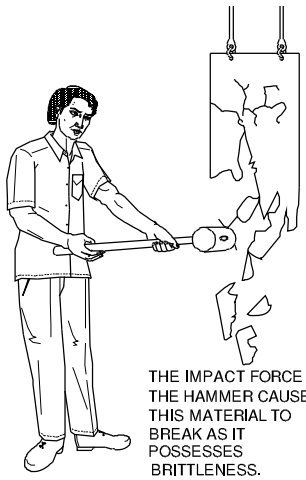
Fig 10



Brittleness (Fig 11)

Brittleness is that property of a metal which permits no permanent distortion before breaking. Cast iron is an example of a brittle metal which will break rather than bend under shock or impact.

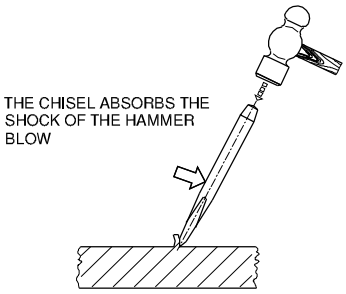
Fig 11



Toughness (Fig 12)

Toughness is the property of a metal to withstand shock or impact. Toughness is the property opposite to brittleness. Wrought iron is an example of a tough metal.

Fig 12



Tenacity

The tenacity of a metal is its ability to resist the effect of tensile forces without rupturing. Mild steel, Wrought Iron and copper are some examples of tenacious metals.

Elasticity

Elasticity of a metal is its power of returning to its original shape after the applied force is released. Properly heat-treated spring is a good example for elasticity.

Material science - Introduction of iron and cast iron

Ferrous Metals

Metals which contain iron as a major content are called ferrous metals. Ferrous metals of different properties are used for various purposes.

Introduction of Iron, Cast Iron, wrought Iron and steel

The ferrous metals and alloys used commonly are:

- Pig-iron
- Cast Iron
- Wrought Iron
- Steels and Alloy steels

Different processes are used to produce iron and steel.

Pig-iron (Manufacturing process)

Pig-iron is obtained by the chemical reduction of iron ore. This process of reduction of the iron ore to Pig-iron is known as SMELTING.

The main raw materials required for producing Pig-iron are:

- Iron ore
- Coke
- Flux

Iron ore

The chief iron ores used are:

- magnetite
- hematite
- limonite
- carbonite.

These ores contain iron in different proportions and are naturally available.

Coke

Coke is the fuel used to give the necessary heat to carry on the reducing action. The carbon from the coke in the form of carbon monoxide combines with the iron ore to reduce it to iron.

Flux

This is the mineral substance charged into a blast furnace to lower the melting point of the ore, and it combines with the non-metallic portion of the ore to form a molten slag.

Limestone is the most commonly used flux in the blast furnace.

Properties and use of Pig-iron

Pig-iron is, therefore, refined and remelted and used to produce other varieties of iron and steel.

Cast Iron (Manufacturing process)

The pig-iron which is tapped from the blast furnace is the crude form of raw material for the cupola, and should be further refined for making castings. This refining is carried out in the cupola furnace which is a small form of a blast furnace.

Generally cupolas are not worked continuously like blast furnaces but are run only as and when required.

Cast Iron (Types)

Cast iron is an alloy of iron, carbon and silicon. The carbon content ranges from 2 to 4%.

Types of cast iron

The following are the types of cast iron.

- Grey cast iron
- White cast iron
- Malleable cast iron
- Nodular cast iron

Grey cast iron

This is widely used for the casting of machinery parts and can be machined easily.

Machine base, tables, slideways are made of cast iron because it is dimensionally stable after a period of aging.

Because of its graphite content, cast iron provides an excellent bearing and sliding surface.

The melting point is lower than that of steel and as grey cast iron possesses good fluidity, intricate casting can be made.

Grey cast iron is widely used for machine tools because of its ability to reduce vibration and minimize tool chatter.

Grey cast iron, when not alloyed, is quite brittle and has relatively low tensile strength. Due to this reason it is not used for making components subjected to high stress or impact loads.

Grey cast iron is often alloyed with nickel, chromium, vanadium or copper to make it tough.

Grey cast iron is weldable but the base metal needs preheating.

White cast iron

This is very hard and is very difficult to machine, and for this reason, it is used in components which should be abrasion-resistant.

White cast iron is produced by lowering the silicon content and by rapid cooling. When cooled in this manner, it is called chilled cast iron.

White cast iron cannot be welded.

Malleable cast iron

Malleable cast iron has increased ductility, tensile strength and toughness when compared with grey cast iron.

Malleable cast iron is produced from white cast iron by a prolonged heat-treatment process lasting for about 30 hours.

Nodular cast iron

This is very similar to malleable cast iron. But this is produced without any heat treatment. Nodular cast iron is also known as: **Nodular Iron - Ductile Iron - Spheroidal Graphite Iron**

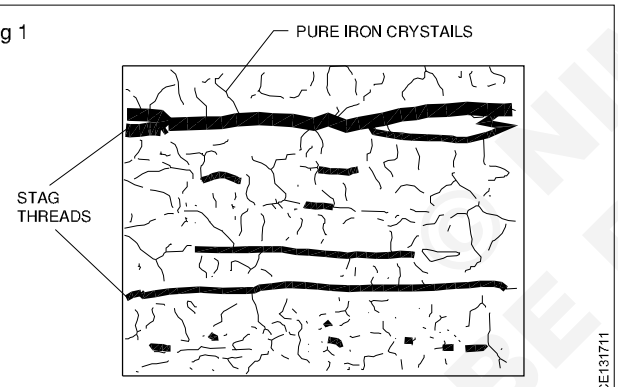
This has good machinability, castability, resistance to wear, low melting point and hardness.

Malleable and nodular castings are used for machine parts where there is a higher tensile stress and moderate impact loading. These castings are less expensive and are an alternative to steel castings.

Wrought Iron (Manufacturing process) (Fig 1)

Wrought iron is the purest form of iron. The analysis of Wrought iron shows as much as 99.9% of iron. (Fig 1) When heated, wrought iron does not melt, but only becomes pasty and in this form it can be forged to any shape.

Fig 1



MICROSTRUCTURE OF WROUGHT IRON

Modern methods used to produce wrought iron in large quantities are the

– puddling process

– aston or Byers process

Steel

This is pure iron. Carbon content is more. Due to excessive carbon it is harder and tougher. Carbon content is from 0.15 to 1.5%. Besides there are other impurities like sulphur, phosphorous etc. are there which cannot be separated. This is hardened and tempered by heating it to a definite temperature and cooling it in oil or water.

The following methods are adopted for making different types of steel:

1 Cementation process 2 Crucible process

3 Bessemer process 4 Open hearth process

5 Electro thermo process 6 High frequency process.

Types of steel

Main two types of steel are:

1 Plain steel

2 Alloy steel

1 Plain steel. In this carbon and iron are mixed. According to the percentage of carbon plain steels are classified as:

A Low carbon steel

B Medium carbon steel

C High carbon steel

A Low carbon steel: It is also called mild steel. In this. the percentage of carbon is from 0.15%to0.25%. Due to less quantity of carbon is sufficiently soft and tolerates the strain. It can be put in different shapes through forging and rolling. This is not very hard or strong. This cannot be hardened or tempered by ordinary methods. Nuts, bolts, rivets, sheets, wires, T-iron and angle iron etc. are made out of it.

B Medium carbon steel: The carbon content is from 0.25% to 0.5%. Due to excess of carbon, it is harder and tougher than mild steel. The tenacity is more. This can be hardened or tempered. Various things are made by forging and rolling. This is used for making high tensile tubes, wires, agricultural implements, connecting rods, cam shafts, spanners, pulleys etc.

C High carbon steel: It has carbon content from 0.5% to 1.5%. It is very hard and wears least. This can be hardened by heat treatment. This can neither be cast nor rolled. This is very hard and tough. It acquires permanent magnetic properties. This is used for making pointed tools, springs, pumps, files, cutleries, cold chisels press die etc.

2 Alloy Steel

When the steel is mixed with other metals like vinoleum, manganese tungsten etc., it is called an alloy steel. Alloy steel has properties of its ingredients.

Types of Alloy Steel

Two types of alloy steel are:

A Low alloy steel

B High alloy steel

A Low Alloy steel: Besides carbon other metals are in lesser quantity. Its tensile strength is more. The welding can work on it. This can also be hardened and tempered. It is used in manufacturing various parts of an aeroplane and cam shaft etc.

B High Alloy Steel: Besides carbon it has a high percentage of the metals higher than low steel alloy. This is classified into following types:

a High Speed Steel: It is also called high tungsten alloy steel because it has more quantity of tungsten. According to the quantity of tungsten it is classified into three types:

1 Tungsten 22%, Chromium4%, Vanadium 1%

2 Tungsten 18%, Chromium 4%, Vanadium 1%

3 Tungsten 14%, Chromium 4%, Vanadium 1%

Cutting tools are made out of it because it is very hard but becomes soft at low critical temperature. This temperature is raised out of cutting process of tool, then the cutting tool becomes useless and is unfit for work. But due to high percentage of tungsten it keeps working upto high temperature. It is used for cutting tools, drills, cutters, reamers, hacksaw blades etc.

b Nickel Steel: In this 0.3% carbon and 0.25 to 0.35% nickel is present. Due to nickel its tensile strength, elastic limit and hardness is increased. It does not catch rust. Its cutting resistance increases 6 times more than plain carbon and steel due to 0.35% nickel present in it. This is used for making rivets, pipes, axle shafting, parts of buses and aeroplanes. If 5% of cobalt is mixed with 30-35% nickel, it becomes invar steel. It is mainly used for making precious instruments.

c Vanadium Steel: It contains 1.5% carbon 12.5% tungsten, 4.5% chromium, 5% vanadium and 5% cobalt. Its elastic limit, tensile strength and ductility is more. It has strength to bear sharp jerks. It is mainly used to manufacture of tools.

d Manganese Steel: It is also called special high alloy steel. It contains 1.6 to 1.9% of manganese and 0.4 to 0.5% carbon. It is hard and less wear. It is not affected by magnet. It is used in grinders and rail points etc.

e Stainless Steel: Along with iron it contains 0.2 to 90.6% carbon, 12 to 18% chromium, 8% nickel and 2% molybdenum. It is used for making knives, scissors, utensils, parts of aeroplane, wires, pipes and gears etc.

Properties of stainless steel:

1 Higher corrosion resistance

2 Higher cryogenic toughness

3 Higher work hardening rate

4 Higher hot strength

5 Higher ductility

6 Higher strength and hardness

7 More attractive appearance

8 Lower maintenance

f Silicon Steel: It contains 14% of silicon. Its uses are multifarious according to the percentage of silicon. 0.5% to 1% silicon, 0.7 to 0.95% manganese mixture is used for construction work. 2.5 to 4% silicon content mixture is used for manufacturing electric motors, generators, laminations of transformers. In chemical industries 14% silicon content mixture is used.

g Cobalt Steel: High carbon steel contains 5 to 35% cobalt. Toughness and tenacity is high. It has magnetic property therefore used to make permanent magnets.

Material science - Difference between iron & steel, alloy steel and carbon steel

Difference between iron and steel:

S.No	Basic distinction	Iron	Steel
1	Formation	Pure substance	Made up of iron and carbon
2	Types	Cast iron, Wrought iron and steel	Carbon steel and alloy steel
3	Rusting	Quickly gets oxidised and result in rust	Have different elements that protect from rusting
4	Surface	Its surface is rusty	Its surface stays shiny
5	Usage	Used in buildings,tools and automobiles	Used in buildings, cars, railways and automobiles
6	Existence	Available in nature	Has to be formed

Steel Plants in India

S.No	Name of the Steel plant	State
1	Tata Iron	Bihar
2	Indian Iron Steel	West Bengal
3	Vishweshvaraiah Iron Steel	Karnataka
4	Bhilai Steel Plant	Chhattisgarh
5	Durgapur Steel Plant	West Bengal
6	Alloy Steel Plant (Durgapur)	West Bengal
7	Bokaro Steel Plant	Bihar
8	Rourkela Steel Plant	Orissa
9	Salem Steel Plant	Tamilnadu
10	Visakhapatnam Steel Plant	Andhra Pradesh

Comparison of the Properties of Cast Iron, Mild Steel and steel

Property	Cast Iron	Mild Steel	Steel
Composition	Carbon contents from 2 to 4.5%	Carbon contents from 0.1 to 0.25%	Carbon contents from 0.5 to 1.7%
Strength	- High compressive strength - Poor tensile strength - Poor shearing strength	- Moderate compressive strength - Moderate tensile strength - High shearing strength	- High compressive strength - High tensile strength - High shearing strength
Malleability	Poor	High	High
Ductility	Poor	High	High
Hardness	Moderately hard and can be hardened by heating to hardening temperature and quenching	Mild	Hard
Toughness	Possesses poor toughness	Very tough with carbon content	Toughness varies
Brittleness	Brittle	Malleable	Malleable
Forgeability	Cannot be forged	Can be forged	Can be forged
Weldability	Cannot be welded with difficulty	Can be welded very easily	Can be welded
Casting	Can be easily cast	Can be cast but not easily	Can be cast
Elasticity	Poor	High	High

	Ferrous metals		Non Ferrous metals
1	Iron content is more	1	Iron content is missing
2	The melting point is high	2	The melting point is low.
3	This is of brown and black colour	3	This is of different colours
4	This catches rust	4	This doesn't catch rust.
5	This can be magnetised	5	This cannot be magnetised
6	This is brittle in cold state.	6	This becomes brittle in hot state.

Difference between cast Iron and steel

	Cast Iron	Steel
1	Carbon content is high	Carbon content is less
2	Carbon is in free state	Carbon is mixed
3	Melting point is low	Melting point is high
4	It cannot be magnetised	It can be magnetised
5	Because it is brittle, it cannot be forged	In can be forged
6	It rusts with difficulty	It rusts quickly
7	It cannot be welded	It can be welded

Difference between metals and non-metals

Metals	Non Metals
Shiny	Dull
Usually good conductors of heat and electricity	Usually poor conductors of heat and electricity
Most are ductile	Not ductile
Opaque (opposite of 'transparent')	Transparent when as a thin sheet
Most are malleable	Usually brittle when solid
Form alkaline oxides	Form acidic oxides
Sonorous (make a bell -like sound when struck)	Not sonorous
Usually have 1-3 valence electrons	Usually have 4-8 valence electrons
Most corrode easily	
Usually high melting point (usually solid at room temperature except for mercury)	

Difference between Carbon steel and alloy steel

	Carbon Steel	Alloy Steel
1	Melting point is low	Melting point is high
2	Easy to work	Hard to work
3	Uniform hardness is not obtained	Uniform hardness is achieved
4	Cutting tool blunt at high temperature during operation	Cutting tool does not blunt at high temperature during operation
5	Rust easily	Does not rust easily
6	Corrosion in acid	Does not corrosion in acid
7	Magnetically attracted	Not attractive by magnetism
8	Low cost	High cost

Material science - Properties and uses of timber

Properties and uses of timber

General properties

Timber should have the following properties

- Straight fibres.
- Silky lustre when planed.
- Uniform colour.
- Regular annual rings.
- Heaviness.
- Firm adhesion of fiber and compact modulary rays.
- Sweet smell.
- It should be free from loose or dead knots and shakes.
- The surface should not clog the teeth of the saw on cutting but should remain bright.

Classification

- Timbers are classified as
  - a Softwood
  - b hardwood

Softwood timber

- Usually all trees with needle leaves of softwood and those with broad leaves are of hard-wood.
- The wood contains resins and turpentines.
- The wood has a fragrant smell.
- Fibres are straight.
- Texture is soft and regular.
- Tough for resisting tensile stresses.
- Weak across the fibres.
- Annual rings are distinct, having one side soft, porous and light coloured. The other side is dense and dark.
- The general colour of the wood is pale tinted or light such as pine spruce, fir, ash, kail, deodar etc.

Properties of hardwood

- The wood generally contains a large percentage of acid.
- It is brightly coloured.
- Annual rings are not distinct.
- It is difficult and hard to work with.
- It resists shearing stress.
- Fibre are overlapped.
- The general colour is dark brown such as oak, walnut, teak, mahagony, sishim, babul, sal etc.

Uses

Soft timber

- Because of its cheapness it is used for low grade furniture, doors and windows for cheap type of houses.
- Used as fuel.
- Some timbers are used for baskets and mat making.
- The bark is used as garment is some places.

Hard timber

- Used for high quality furniture such as chairs, tables, sofas, dewans, beds, etc.
- Used for door, window frames for high quality houses as they can take good polish and painting finish.
- Used for manufacturing katha.

Wood as an electrical insulator

Wood is impregnated with oil or other substance, for use as insulator.

Example

It is used in electrical machine windings, as slot wedges.

Mass, Weight, Volume and Density - Mass, volume, density, weight and specific gravity

Mass

Mass of a body is the quantity of matter contained in a body. The unit of mass in F.P.S system is pound (lb), in C.G.S. system gram (gr) and in M.K.S and S.I systems kilogram (kg). 1ton which is 1000 kg is also used sometimes. The conversion factor is 1000. Three decimal places are shifted during conversion.E.g.1 ton =1000 kg 1g = 1000mg.

m - mass of a body

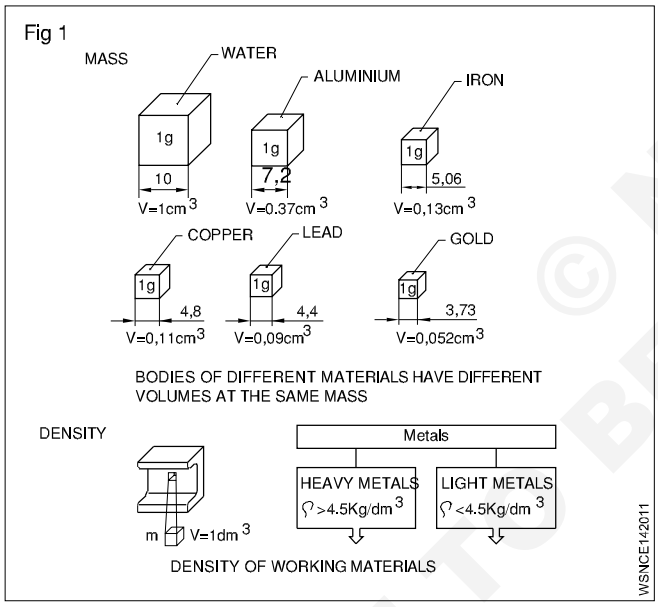
g - acceleration due to gravity in metre/sec^2 = 9.81 m/ sec^2

V - volume of the body

ρ - density (pronounced as `rho')

W or FG - weight or weight force

Mass (Fig 1)



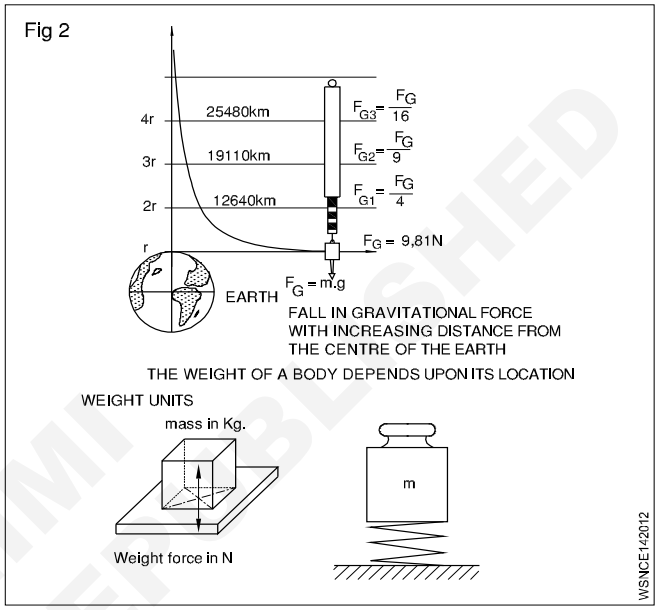
Density

Density is the mass of a body per unit volume. Hence its unit will be gr/cm^3 or kg/dm^3 or ton/m^3.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{m}{v} = \rho$$

Weight (Fig 2)

Weight is the force with which a body is attracted by the earth towards its centre. It is the product of the mass of the body and the acceleration due to gravity. The weight of a body depends upon its location.



weight = *W* or *FG* = mass x gravitational force
= m x g

System	Absolute unit	Derived unit	Conversion
F.P.S. system	1 poundal	1 Lb wt	32.2 poundals (1 lb x 1 ft/sec^2 = 1 pound)
C.G.S. system	1 dyne 1 gr x 1 cm/sec^2	1 Gr.wt	981 dynes
M.K.S.	Newton	1 kg.wt	1 Newton =
S.I.system	Newton	Newton	1 kg x 1 m/sec^2

1 kg.wt = 9.81 Newton (approximately 10N) **1 Newton = 10^5 dynes.**

Difference between mass and weight

S. No	Mass	Weight
1	Mass is the quantity of matter in a body (ie) measurement of matter in a body	Weight is measure of amount of force acting on mass due to acceleration due to gravity
2	It does not depend on the position or space	It depends on the position, location and space
3	Mass of an object will not be zero	Weight of an object will be zero if gravity is absent
4	It is measured using by physical balance	It is measured using by spring balance
5	It is a scalar quantity	It is a vector quantity
6	When immersed in water mass does not change	When immersed in water weight will change
7	The unit is in grams and kilogram	The unit is in kilogram weight, a unit of force

Mass and weight are different quantities.

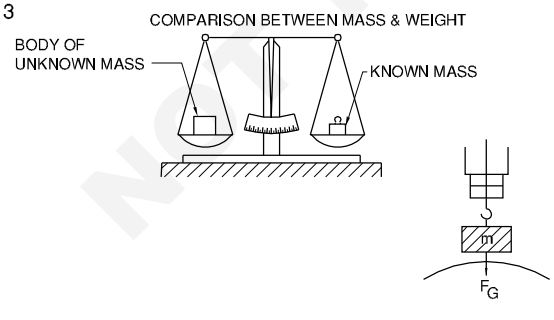
Mass of a body is equal to volume x density.

Weight force is equal to mass x acceleration due to gravity.

Weight , Density and Specific gravity

It is now seen that the mass of a substance is measured by its weight only without any reference to volume. But if equal weights of lead & aluminium, are compared the volume of lead is much smaller than volume of aluminium. So we can now say that lead is more dense than aluminium,. i.e In other words the density of lead is greater than aluminium. (Fig 3 & 4)

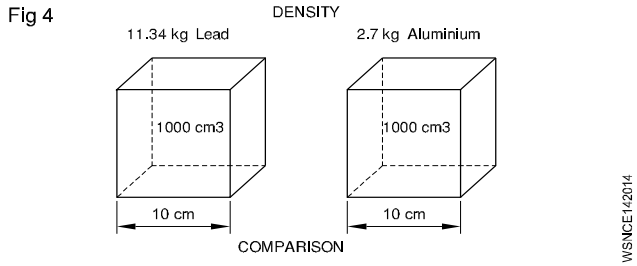
Fig 3



The relation of mass and volume is called density.

The density expresses the mass of volume E.g. 1 dm^3 of water has the mass of 1kg - thus the density of 1kg/dm^3 (Fig 2)

Fig 4



Unit

The density is measured as below

MKS/SI= Kg/m^3, CGS = 1 gm/cm^3 FPS = lbs/c ft

	Solids	gm/cc	Liquids	gm/cc
1	Aluminum	2.7	Water	1.00
2	Lead	11.34	Petrol	0.71
3	Cast iron	6.8 to 7.8	Oxygen	1.43
4	Steel	7.75 to 8.05	Diesel Oil	0.83

The specific gravity of a substance is also called its relative density.

Formula

Specific gravity (or) Relative density $= \frac{\text{Density of the substance}}{\text{Density of the water at } 4^\circ C}$

$= \frac{\text{Mass of any volume of a substance}}{\text{Mass of an equal volume of water at } 4^\circ C}$

Comparison Between Density And Specific Gravity (Relative Density)

Density	Relative density or Specfic gravity
Mass per unit volume of a substance is called its density	The density of substance to density of water at 4°C is its relative density
Its unit is gm per cu cm; lbs per cu.ft and kg/cubic meter	It has no unit of measure-ment simply expressed in a number
Density $= \frac{\text{Mass}}{\text{Volume}}$	Relative density $= \frac{\text{Density of the substance}}{\text{Density of water at } 4^\circ C}$

	Solids	Sp.gy	Liquids	Sp.gy
1	Aluminium	2.72	Petrol	0.71
2	Lead	11.34	Battery acid	1.2 to 1.23
3	Cast iron	6.8 to 7.8	Water	1.00
4	Steel	7.82	Diesel Oil	0.83

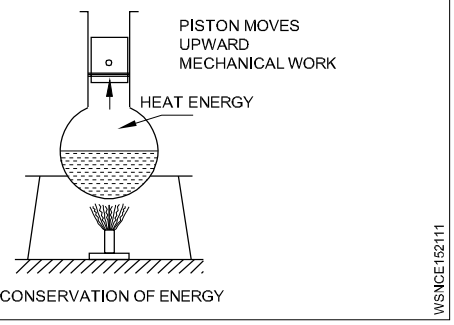
From the above table, we can calculate the weight of any given volume of a substance (say Diesel oil) in any units provided we know the specific gravity of the substance. Also vice-versa for volume of density is known.

Heat & Temperature and Pressure - Concept of heat and temperature, effects of heat, difference between heat and temperature, boiling point & melting point of different metals and non-metals

Heat

It is a form of energy. Heat energy can be transformed into other forms of energies. Heat flows from a hotter body to a colder body. (Fig 1)

Fig 1



CONSERVATION OF ENERGY

Units of heat

Calorie: It is the quantity of heat required to raise the temperature of 1 gram of water through 1°C.

BTHU: It is the quantity of heat required to raise 1 lb of water through 1°F. (British thermal unit).

C.H.U; It is the quantity of heat required to raise 1 lb of water through 1°C.

Joule : S.I. Unit (1 Calorie = 4.186 joule)

Effects of heat

- Change in temperature
- Change in size
- Change in state
- Change in structure
- Change in Physical properties

Specific heat

The quantity of heat required to raise the temperature of one gm of a substance through 1^0C is called specific heat. It is denoted by the letter 's'.

Specific heat of water	= 1
Aluminium	= 0.22
Copper	= 0.1
Iron	= 0.12

Thermal capacity:

It is the amount of heat required to raise the temperature of a substance through 1^0C is called the thermal capacity of the substance.

Thermal capacity = ms calories.

Calorific value: The amount of heat released by the complete combustion of unit quantity of the fuel (Mass or volume) is known as calorific value of fuels.

Water equivalent

It is the mass of water which will absorb the same amount of heat as the given substance for the same temperature rise. Water equivalent = Mass of the substance x specific heat of the substance.

Therefore water equivalent = ms

Types of heat

1 Sensible heat

2 Latent heat

1 Sensible heat

Sensible heat is the heat absorbed or given off by a substance without changing its physical state. It is sensible and can be absorbed by the variation of temperature in the thermometers.

2 Latent heat

The heat gained or given by the substance during a change of state (from solid to liquid to gas) is called latent heat or hidden heat. The heat absorbed or given off does not cause any temperature change in the substance.

Types, 1. Latent heat of fusion of solid

2. Latent heat of vaporisation of solid.

1 Latent heat of fusion of solid

The amount of heat required per unit mass of a substance at melting point to convert it from the solid to the liquid state is called latent heat of fusion of solid. Its unit is cal/gram.

Latent heat of fusion of ice

The amount of heat required to convert per unit mass of the ice into water at 0^0C temperature is called latent heat of fusion of ice.

Latent heat of fusion of ice(L) = 80 cal/gram

2 Latent heat of vaporisation of liquid

The amount of heat required to vaporise a unit mass of liquid at its boiling point is called latent heat of vaporisation.

Latent heat of vaporisation of water or latent heat of steam

The amount of heat required to convert into steam of a unit mass of water at its boiling point (100^0C) is called latent heat of vaporisation of water or latent heat of steam.

Latent heat of steam(L) = 540 cal/gram

Temperature

It is the degree of hotness or coldness of a body. The temperature is measured by thermometers.

Difference between heat and temperature

Heat	Temperature
1 It is a form of energy.	This tells the state of heat.
2 Its unit is calorie.	Its unit is degree.
3 Heat is measured by calorimeter.	Temperature is measured by thermometer.
4 By adding quantity of heat of two substances their total heat can be calculated.	By adding two temperatures we cannot find the temperature of the mixture.
5 By heating a substance the quantity of heat is increased regardless of increase in temperature.	Two substances may read the same temperature though they might be having different amount of heat in them.

Boiling point

Any substance starts turning into a gas shows the temperature at which it boils this is known as the boiling point. The boiling point of water is 100^0C.

Melting point

The temperature at which any solid melts into liquid or liquid freezing to solid is called the melting point of substance. ` The melting point of ice is 0^0C.

List of melting point and boiling point of metals and Non -metals

Metals and Non-metals	Melting point °C	Boiling point °C
Aluminium	660.25	2519
Argon	-189.19	-185.85
Arsenic	817	614
Barium	729	1897
Beryllium	1287	2469
Bromine	-7.1	58.8
Cadmium	321.18	767
Calcium	839	1484
Carbon (diamond)	3550	4827
Carbon (graphite)	3675	4027
Chlorine	-100.84	-34.04
Cobalt	1495	2927
Copper	1084.6	2562
Gold	1064.58	2856
Helium	-	-268.93
Hydrogen	-259.98	-252.87
Iodine	113.5	184.3
Iridium	2443	4428
Iron	1535	2861
Lead	327.6	1749
Lithium	180.7	1342
Magnesium	650	1090

Metals and Non-metals	Melting point °C	Boiling point °C
Manganese	1246	2061
Mercury	-38.72	357
Molybdenum	2617	4639
Nickel	1453	2913
Nitrogen	-209.86	-195.79
Oxygen	-226.65	-182.95
Phosphorus (white)	44.1	280
Plutonium	640	3228
Potassium	63.35	759
Radium	700	1737
Silicon	1410	3265
Silver	961	2162
Sodium	98	883
Sulfur	115.36	444.6
Tin	232.06	2602
Titanium	1660	3287
Tungsten (wolfram)	3422	5555
Uranium	1132	4131
Zinc	419.73	907

Heat & Temperature and Pressure - Scales of temperature, celsius, fahrenheit, kelvin and conversion between scales of temperature

Temperature Scales

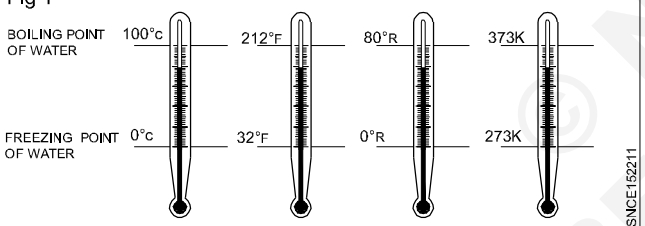
Temperature is calibrated between two fixed reference points namely the freezing point of water, and the boiling point of water. These two fixed points on different temperature scales are:

Scale	Freezing point	Boiling point
Centigrade (°C)	0°C	100°C
Fahrenheit (°F)	32°F	212°F
Kelvin (K)	273°K	373°K
Reaumur (°R)	0°R	80°R

Heat is a form of energy. Temperature is the degree of hotness or coldness of a body. The relationship for conversion from one temperature scale to the others is

$$\frac{^0R}{80} = \frac{^0C}{100} = \frac{^0K - 273}{100} = \frac{^0F - 32}{180}$$

Fig 1



1 Convert 0^0C into ^{0}F

$$\frac{^0F - 32}{180} = \frac{^0C}{100}$$

$$^0F - 32 = \frac{^0C}{100} \times 180$$

$$^0F - 32 = \frac{0}{100} \times 180$$

°F = 0 + 32

= 32^0F

0^0C = 32^0F

2 Convert -40^0C into ^{0}F

$$\frac{^0F - 32}{180} = \frac{^0C}{100}$$

$$^0F - 32 = \frac{^0C}{100} \times 180$$

$$^0F - 32 = \frac{-40}{100} \times 180$$

F - 32 = -72

°F = -72 + 32

= -40^0F

-40^0C = -40^0F

3 Convert 37^0C into K

$$\frac{^0C}{100} = \frac{^0K - 273}{100}$$

°K-273 = C

°K = C + 273

°K = 37 + 273

= 310 K

37^0C = 310K

4 Convert 70^0C into Reaumur

$$\frac{^0C}{100} = \frac{^0R}{80}$$

$$^0R = \frac{C}{100} \times 80$$

$$^0R = \frac{70}{100} \times 80 = 56$$

70^0C = 56^0R

5 Convert -25^{0}F into ^{0}C

$$\frac{^0C}{100} = \frac{^0F - 32}{180}$$

$$\frac{°C}{100} = \frac{-25 - 32}{180}$$

$$^0C = \frac{-57}{180} \times 100$$

$$^0C = \frac{-285}{9} = -31.66$$

- 25^{0}F = -31.7^{0}C

6 Convert 98.6^{0}F into ^{0}C

$$^0C = \frac{^0F - 32}{180} \times 100$$

$$^0C = \frac{98.6 - 32}{180} \times 100$$

$$= \frac{66.6}{180} \times 100$$

$$= \frac{6660}{180} = 37^0C$$

98.6^{0}F = 37^{0}C

Assignment

Convert the following

1 10.5^{0}C = ___^{0}F

2 40^{0}C = ___^{0}F

3 60^{0}C = ___^{0}F

4 80^{0}C = ___^{0}F

5 105^{0}C = ___^{0}F

6 100^{0}C = ___^{0}F

7 -80^{0}C = ___^{0}F

8 200^{0}C = ___^{0}F

9 605^{0}C = ___^{0}F

10 1250^{0}C = ___^{0}F

11 77^{0}F = ___^{0}C

12 20^{0}F = ___^{0}C

13 428^{0}F = ___^{0}C

14 -210^{0}F = ___^{0}C

15 88^{0}F = ___^{0}C

16 110^{0}F = ___^{0}C

17 72^{0}F = ___^{0}C

18 50^{0}F = ___^{0}C

19 900^{0}F = ___^{0}C

20 72^{0}R = ___^{0}C

21 143^{0}C = ___K

22 373^{0}K = ___^{0}C

23 746^{0}K = ___^{0}F

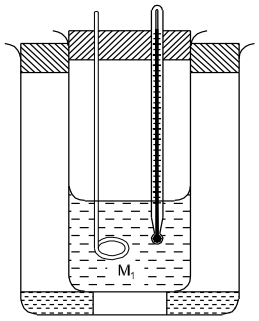
24 At what temperature will the reading of a fahrenheit thermometer be double of a centigrade one.

Heat & Temperature and Pressure - Temperature measuring instruments, types of thermometer, pyrometer and transmission of heat - Conduction, convection and radiation

Measuring heat energy

Energy can be released in chemical reactions as light, sound or electrical energy. But it is most often released as heat energy. This allows us to easily measure the amount of heat energy transferred.

Fig 1



SIMPLE CALORIMETER

WSNCE152311

The apparatus used to measure the amount of heat by mixer method is called calorimeter. It is nothing but cylindrical shaped vessel and a stirrer made out of mostly copper.

In a calorimeter when the hotter solid/liquid substance are mixed with the cooler solid/liquid substances, heat transfer takes place until both substances reach the same temperature. By the same time calorimeter also reaches the same temperature. By mixing rule,

$$\begin{bmatrix}\text{Loss of heat}\\ \text{by solid/}\\ \text{liquid}\end{bmatrix} = \begin{bmatrix}\text{Heat absorbed by}\\ \text{solid / liquid}\\ \text{substance}\end{bmatrix} + \begin{bmatrix}\text{Heat absorbed}\\ \text{by calorimeter}\end{bmatrix}$$

Measurement

Temperature is generally measured in degrees Celsius. In this system the freezing point of water is defined as 0°C and the boiling point of water is defined as 100°C. The Kelvin temperature scale begins from absolute 0. i.e. 273°. The temperature intervals are the same.

∴ 273K = 0°C, 20°C = 273K + 20°C = 293K.

Instruments

The instruments used to measure and read temperature takes into account changes in the properties of materials, electrical phenomena incandescence, radiation and melting.

Thermometer

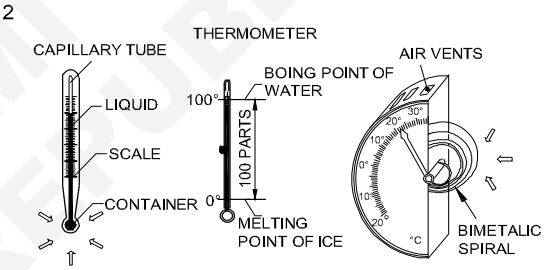
Types of thermometer

- Forehead strips
- Wearable thermometers
- Pacifier thermometers
- Ear thermometers (tympanic)
- Forehead thermometers (temporal)
- Digital thermometers
- Mom's hand or lips

They are based on the principle that liquids and solids expand when they are subjected to heat. Mercury and alcohol expand uniformly. When heat is applied the volume of the liquid increases and the liquid rises in the capillary tube integral with the container. Mostly mercury is used in this type of thermometers because of its properties (Shiny and will not adhere to the glass tubes and we can measure up to 300^0C.

The bimetal thermometer consists of metals with different coefficient of expansion. The bimetal is twisted into a spiral which curls when the temperature rises.

Fig 2



BIMETALIC THERMOMETER (HALF THE CASTING REMOVED)

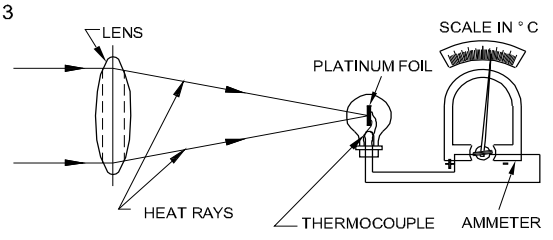
WSNCE152312

Pyrometer

Thermoelectric pyrometer is based on the principle that the soldering point between the wires of different metals, when heated a contact voltage is generated. The voltage depends upon the temperature difference between the hot measuring point and the cold end of the wire. Thermocouple elements are constructed of copper and Constant (up to 600°C) or of platinum and platinum-rhodium (up to 1600°C)

Radiation pyrometers are used to measure temperatures of red hot metals up to 3000°C. These concentrate thermal rays through an optical lens and focus them on to a thermo element. The scale of the ammeter is calibrated in degrees Celsius or Kelvin.

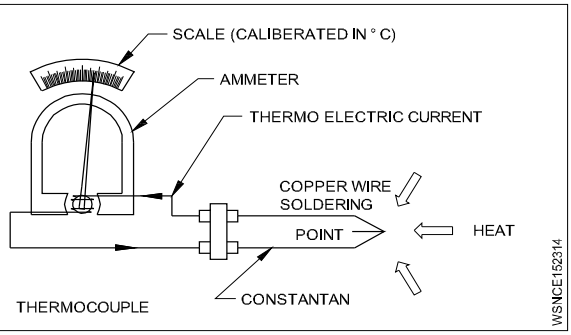
Fig 3



IN THE RADIATION PYROMETER, THERMAL RADIATION IS FOCUSED ON THE SOLDERING POINT OF A THERMOCOUPLE

WSNCE152313

Fig 4



Transmission of Heat

Heat is a form of energy and is capable of doing work. Heat flows from a hot body to a colder body or from a point of high temperature to a point of low temperature. The greater is the temperature difference the more rapidly will be the heat flow. Heat is transmitted in three ways.

1 Conduction

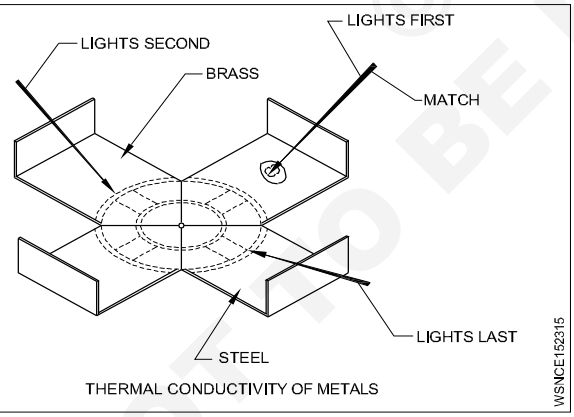
2 Convection

3 Radiation

Conduction

Conduction is the name given to the transmission of heat energy by contact. The heat source is in contact with the Conductor. (metal rod). The rod is in contact with a thermometer. Due to Conduction heat is transferred from the heated end to the free end. In general good electrical conductors are also good heat conductors and good electrical insulators are also good heat insulators. A good heat insulator does not necessarily withstand high temperature.

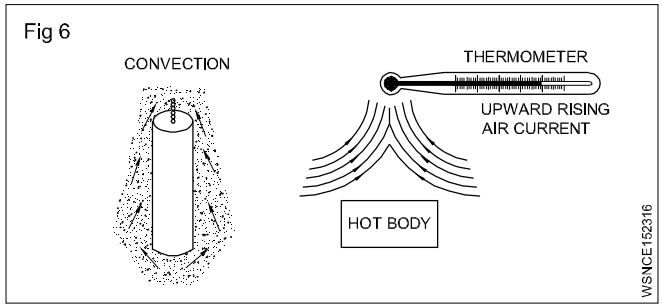
Fig 5



Convection

Convection is the name given to the transmission of heat energy by the up-ward flow. When heated, the fluid (liquid/ gas) becomes less dense and because of its mobility, is displaced upwards, by a similar but colder and more dense fluid. e.g., The domestic hot water system, The cooling system in motor cars.

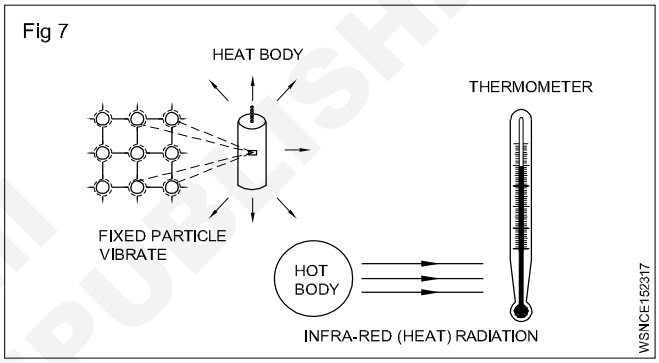
Fig 6



Radiation

Heat is radiated or transmitted from one object to the other in space without actually being in contact, by means of electro-magnetic waves. These waves are similar to light waves and radio waves. They can be refracted by lenses and reflected by mirrors. This radiation is called infrared. It requires no medium to carry the radiation. (e.g) The heat of the sun travels through the space.

Fig 7



Transmission of heat takes place in three ways Conduction, Convection and Radiation.

Expansion due to heat

When a solid, liquid or gaseous substance is heated, it expands and volume is increased. Similarly when it is cooled, it contracts (shrinks) and volume is decreased.

E.g : small gaps are left in between the lines of railway track to allow for expansion during summer. If this is not done, the rails would expand and bend there by causing derailment of trains.

Except a few substances, all solids, liquids and gases expand. For the same amount of heat given, the expansion of liquids is greater than solid and expansion of gas is more than liquid.

Volume of water is reducing while heating from 0^0C to 4^0C. After that volume is increasing. The data at 4^0C of water will be taken as reference point for any calculations relating with water.

Heat & Temperature and Pressure - Co-efficient of linear expansion and related problems with assignments

Expansion of solids

A solid substance shows the following types of expansion when heated.

1 Linear expansion

2 Superficial expansion

3 Cubical expansion

1 Linear expansion

When a solid is heated, its length increases. This is called linear expansion. It depends upon the material, original length and change in temperature.

Co-efficient of linear expansion

The co-efficient of linear expansion is the change in length per unit original length per degree rise in temperature. It is represented by α (Alpha).

Length of the solid at $t_1{}^0C = l_1$

Length of the solid at $t_2{}^0C = l_2$

Change in Temperature $= t_2 - t_1{}^0C$

Change in length $= l_2 - l_1$

$$\alpha = \frac{l_2 - l_1}{l_1 \times (t_2 - t_1)}$$

$$\alpha = \frac{l_2 - l_1}{l_1 t} [t_2 - t_1 = t]$$

$$\left.\begin{matrix}\text{Co - efficient of} \\ \text{linear expansion}\end{matrix}\right\} = \frac{\text{Change in length}}{\text{Original length x change in temperature}}$$

Increased length $l_2 - l_1 = \alpha l_1 t$

Final length $l_2 = l_1(1 + \alpha t)$

2 Superficial expansion

When a solid is heated, its area increases is called superficial expansion.

Co-efficient of superficial expansion

The increase in area per unit original area per degree rise in temperature is called co-efficient of superficial expansion. It is represented by β (Beta).

Co-efficient of superficial

Expansion = 2 x linear expansion

$$\beta = 2\alpha$$

3 Cubical expansion

When a solid is heated, its volume increases is called cubical expansion.

Co-efficient of cubical expansion

The increase in volume per unit original volume per degree rise in temperature. It is represented by γ (Gama).

Co-efficient of cubical expansion

= 3 x linear expansion

$$\gamma = 3\alpha$$

Examples

Find the co-efficient of linear expansion. If an 8 metre long metal rod is heated from 30^{0}C to 80^{0}C. So that it may produce an elongation of 0.84 mm.

Initial length (l) = 8m

Increased length = 0.84 mm

Increased temperature(t) = 80 - 30 = 50^{0}C

$$\left.\begin{matrix}\text{Co - efficient of linear} \\ \text{expansion}(\alpha)\end{matrix}\right\} = \frac{\text{Increased length}}{\text{Initial length} \times \text{Increased temp}}$$

$$= \frac{0.84}{8000 \times 50}$$

$$= \frac{0.84}{400000}$$

$$= 2.1 \times 10^{-6} /{}^0C$$

If iron bridge is 100 metre long at 0^{0}C. What will be the length of bridge if the temperature is 40^{0}C and the co-efficient of linear expansion is 12 x 10^{-6} per degree.

Initial length of iron bridge = 100 m

Increased temperature = 40 - 0 = 40^{0}C

$$\left.\begin{matrix}\text{Co - efficient of linear} \\ \text{expansion}(\alpha)\end{matrix}\right\} = \frac{\text{Increased length}}{\text{Initial length} \times \text{Increased temp}}$$

$$12 \times 10^{-6} = \frac{\text{Increased length}}{100 \times 40}$$

$$\text{Increased length} = \frac{12}{1000000} \times 100 \times 40$$

$$= 0.048 \text{ m}$$

Iron bridge at 40^{0}C = 100 + 0.048 = 100.048 m

The length of a metal rod is 100 cm at 30^{0}C and 100.14 cm at 100^{0}C. Calculate the co-efficient of linear expansion and the rod length in 0^{0}C.

Initial length at 30^{0}C = 100 cm

Final length at 100^{0}C = 100.14 cm

Increased length = 0.14 cm

Increased temperature = 100 - 30 = 70^{0}C

$$\left.\begin{matrix}\text{Co-efficient of linear}\\ \text{expansion}(\alpha)\end{matrix}\right\} = \frac{\text{Increased length}}{\text{Initial length} \times \text{Increased temp}}$$

$$= \frac{0.14}{100 \times 70}$$

$$= \frac{14}{100 \times 70 \times 100}$$

$$= \frac{2}{100000}$$

$$= 2 \times 10^{-5}$$

To find the length at 0^0C

$$l_1 = l_0 (1 + \alpha t)$$

$$100 = l_0 (1 + 2 \times 10^{-5} \times 30)$$

$$100 = l_0 (1 + 0.0006)$$

$$l_0 = \frac{100}{1 + 0.0006}$$

Length at 0°C = 99.94 m

Find the change in length of metallic rod 100 cm long, when its temperature is increased from 25^0C to 40^0C and the co-efficient of linear expansion is 10 x 10^{-6} / ^{0}C.

Initial length = 100 cm

Increased temperature = 40 - 25 = 15^0C

Co-efficient of linear expansion (α) = 10 x $10^{-6}/^0$C

$$\left.\begin{matrix}\text{Co-efficient of linear}\\ \text{expansion}(\alpha)\end{matrix}\right\} = \frac{\text{Increased length}}{\text{Initial length} \times \text{Increased temp}}$$

$$10 \times 10^{-6} = \frac{\text{Increased length}}{100 \times 15}$$

Increased length = 10 x 10^{-6} x 100 x 15

$$= \frac{10 \times 100 \times 15}{1000000}$$

$$= \frac{15}{1000} = 0.015\text{cm}$$

Find out the temperature that the rod will extend by 0.54 mm in linear direction when a piece of metal rod is 2.5 metre long in 20^0 C and the co-efficient of linear expansion is 10.4 x 10^{-6} per degree centigrade.

Initial length = 2.5 m = 2500 mm

Increased length = 0.54 mm

Initial temperature = 20^0C

Co-efficient of linear expansion (α) = 10.4 x 10^{-6}

$$\left.\begin{matrix}\text{Co-efficient of linear}\\ \text{expansion}(\alpha)\end{matrix}\right\} = \frac{\text{Increased length}}{\text{Initial length} \times \text{Increased temp}}$$

$$10.4 \times 10^{-6} = \frac{0.54}{2500 \times \text{Increased temp}}$$

$$\text{Increased temperature} = \frac{0.54}{2500 \times 10.4 \times 10^{-6}}$$

$$= \frac{0.54 \times 1000000}{2500 \times 10.4}$$

$$= \frac{5400}{260} = 20.77\ ^0\text{C}$$

Final temperature = 20 + 20.77

= 40.77^0C

Assignment

Co-efficient of linear expansion

1 Calculate the co-efficient of linear expansion of rod. If rod is found to be 100m long at 20^0C and 100.14m long at 100^0C.

2 Find the change in length if the co-efficient of linear expansion of rod is 0.00024/^{o}C and the temperature of a rod of 3.6m length is raised by 120^0C,

3 Find the change in length if the co-efficient of linear expansion of rod is 0.00024/^{0}C. If the temperature of a rod of 6m length is raised by 120^0C,

4 Find the increase in length 100 cm iron rod if the temperature raise from 40^0C to 90^0C. The co-efficient of linear expansion of the iron is 10x$10^{-6}/^0$C

5 If micrometer reading is standardised at 15^0C. What will be the true reading of the micrometer if the reading taken at 35^0C is 20.20 mm?

The co-efficient of linear expansion of material of micrometer is 11 x $10^{-6}/^0$C.

Basic Electricity - Introduction and uses of electricity, molecule, atom, how electricity is produced, electric current AC,DC their comparison, voltage, resistance and their units

Electricity is a kind of energy. It is the most useful sources of energy which is not visible but its presence can be felt by its effects. Electricity is obtained by conversion of other forms of energy like heat energy, chemical energy, nuclear energy, mechanical energy and energy stored in water etc.,

To understand electricity, one must understand the structure of an atom.

Basically an atom contains electrons, protons and neutrons. The protons and neutrons are located in the centre of an atom and the electrons, a negative electric charge particle revolving around the nucleus in an atom. The proton has a positive charge. Neutrons are neutral and have no charge.

Sources of electricity

Battery

Battery stores electrical energy in the form of chemical energy and it gives power when required. Battery is used in automobiles and electronics, etc.,

Generator

It is a machine which converts the mechanical energy into electrical energy.

When a conductor rotates between a magnetic field using prime mover an emf will be induced. By using this method all types of AC and DC generator - generates power.

E.g. Thermal power station
Hydro power station
Nuclear power station
Wind power station
Solar power station

Thermo couple

If two dissimilar pieces of metals are twisted together and its joined end is heated in a flame, then a potential difference or voltage will be induced across the ends of the wires. Such a device is known as a Thermo couple. Thermo couple is used to measure very high temperature of furnaces.

Effects of electric current

When an electric current flows through a medium, its presence can be felt by its effects, which are given below.

1 Physical effect

Human body is a good conductor. when the body touches the bare current carrying conductor, current flows through the human body to earth and body gets severe shock or cause even death in many cases.

2 Magnetic effect

When an electric current passes through a coil, a magnetic field is produced around it.

E.g. : Electromagnet Motor, Generator, Electric bell

3 Chemical effect

When an electric current passes through an electrolyte, chemical action takes place. Because of that, an electrical energy is stored in a battery as a chemical energy.

E.g.: Electroplating, Cells and battery charging, refining of metals etc.,

4 Heating effect

When an electric current passes through any conductor, heat is produced in the conductor due to its resistance.

E.g. : Electric heater, Electric iron box, Electric lamp, Geyser, Soldering iron, Electric kettles, Electric welding etc.,

5 X-ray and Laser rays effect

When a high frequency voltage is passed through a vacuum tube, a special type of rays come out, which is not visible. These rays are called x-rays. Laser rays also can be produced by electric current.

6 Gas effect

When electrons pass through a certain type of sealed glass shell containing gas, then it emits light rays.

E.g: Mercury vapour lamp, Sodium vapour lamp, Fluorescent lamp, Neon lamp etc.,

Uses of Electricity

1	Lighting	- Lamps
2	Heating	- Heaters, ovens
3	Power	- Motor, fan
4	Traction	- Electromotive, lift, crane
5	Communication	- Telephone, telegraph, radio, wireless
6	Entertainment	- Cinema, radio, T.V.
7	Medical	- x-rays, shock treatment
8	Chemical	- Battery charging, electroplating
9	Magnetic	- Temporary magnets
10	Engineering	- Magnetic chucks, welding, x-rays of welding

Classification

- Static electricity
- Dynamic electricity

STATIC ELECTRICITY

If a dry glass rod is rubbed with silk cloth the glass rod gives out negative electrons, and therefore, becomes positively charged. The silk cloth receives negative electrons and therefore it becomes negatively charged. They acquire the property of attracting small pieces of paper etc. because like charges repel and unlike charges attract each other. The electric charge on the silk cloth is stationary and is called static electricity. This type of electricity cannot be transmitted from one place to another.

DYNAMIC ELECTRICITY

The electrons in motion are called current electricity or electric current. This type of electricity is carried through wires and cables. Therefore, this electricity can be transmitted from one place to another. This type of electricity can be produced by cells, batteries, generators alternators etc.

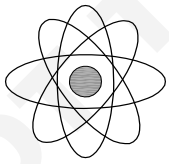
What is the difference between an atom and an element? How are molecules different from atoms? I am often asked these questions in my sessions over and over again and so I finally decided to write a comprehensive post on them. Find answers to all your questions in this section that is designed to help students explore and understand the relationship between atoms, elements, molecules, compounds and mixtures in a manner that is simple and easy to understand.

What is an Atom?

All the matter in the universe is made of tiny particles called atoms. There are 92 different kinds of atoms in nature. These 92 different atoms combine with one another to form different kinds of matter that we see in nature. (Fig 1)

Gold, for example, is made of only gold atoms. When matter is made of only one kind of atom, it is called an element. In the same way, silver is another element which is made of only silver atoms. Because there are 92 different kinds of atoms in nature, there are 92 different kinds of elements. Other examples of an atom are K (potassium) and Fe (iron).

Fig 1



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What is a Molecule?

A molecule is the smallest unit of a chemical compound and it exhibits the same chemical properties of that specific compound. As molecules are made up of atoms jointly held by chemical bonds, they can vary greatly in terms of complexity and size. The oxygen we breathe has a molecular formula O_2. Should we consider this as an element or compound? When two or more atoms of the same elements combine together, we call them Molecules. So, we call O_2 as an oxygen molecule. In the same way, we find hydrogen molecules H_2, chlorine molecules Cl_2 and others in nature.

Types of electric current

- Direct current
- Alternating current

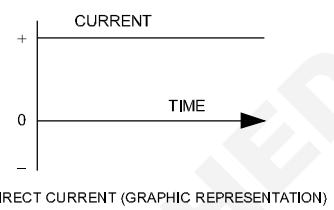
Direct current

In direct current (DC) the direction and magnitude of the current does not change (Fig 2). The steady current flow will be from the positive terminal to the negative terminal. (Fig 3)

Examples

DC Sources : Cells, batteries and DC generators (Fig 3)

Fig 2



DIRECT CURRENT (GRAPHIC REPRESENTATION)

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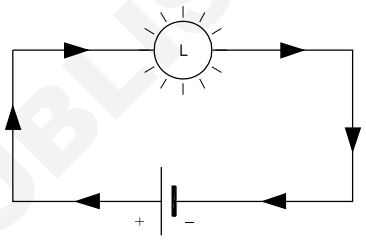
Fig 3



CURRENT FLOWS FROM +VE TERMIINAL TO –VE TERMINAL

DIRECT CURRENT (D.C)

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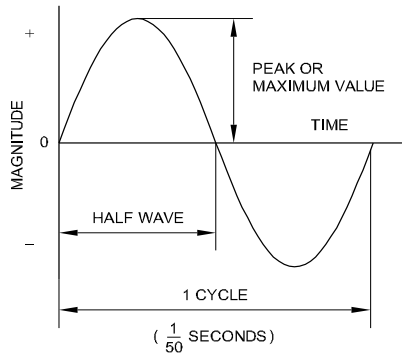
Alternating current (Fig 4)

The current flow will be from the phase terminal to the Neutral terminal. In the alternating current (AC) both the direction and magnitude of the current will be changing at definite intervals of time. The graph shows how an AC current or voltage changes with time. The current increases to the maximum value in one direction, falls to zero and increases to the maximum value in the other (opposite) direction before falling to zero again. Thus a cycle is one complete series of changes. The normal supply frequency is 50 cycles per second.

Fig 4



ALTERNATING CURRENT OR VOLTAGE

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Difference between AC and DC

AC	DC
1 It is generated in the ranges of 6,600 V, 11000 V and 33,000 V.	It is generated up to 6,600 V only
2 Voltage can be stepped up or stepped down by using transformer	It is not possible
3 Transmission cost is less	Cost High
4 Less maintenance	High maintenance
5 Power up to 5,00,000 kw can be generated in a single alternator.	Power up to 10,000 kw can be generated in a single generator
6 AC generator can run at high speeds. So, speed control is not easy.	It can't run at high speeds. Speed control is easy.
7 Slip rings and brushes are used to collect the current.	Commutator and brushes are used to collect the current

Advantages of A.C.

i In transmission there is saving in copper wire.

ii Since there is no spark in A.C. machine there is no interference in Radio sound.

iii This can be produced to maximum voltage i.e. 33000volts.

iv Voltage can be dropped or raised with the help of transformers.

v Its mechanism is simple and cheap.

vi Output is more due to availability of more than one phase.

Disadvantages of A.C.:

i A single phase motor is not self-starter.

ii Due to thin wire in A.C., the voltage drop is more.

iii It cannot be used for electroplating and in charging secondary cells.

iv The speed of motors run by it is difficult to change.

v There is danger to security due to high voltage.

Electrical terms and units

Quantity of electricity

The strength of the current in any conductor is equal to the quantity of electrical charge that flows across any section of it in one second. If 'Q' is the charge and 't' is the time taken

then $I = \frac{Q}{t}$ $\quad Q = I \times t$

The SI unit of current is coulomb. Coulomb is equivalent to the charge contained in nearly 6.24×10^{18} electrons.

Coulomb

In an electric circuit if one Ampere of current passes in one second, then it is called one coulomb. It is also called ampere second (As). Its larger unit is ampere hour (AH)

1 AH = 3600 As (or) 3600 coulomb

Electro motive force (EMF)

It is the force which causes to flow the free electrons in any closed circuit due to difference in electrical pressure or potential. It is represented by 'E.' Its unit is Volt.

Potential difference (P.D)

This is the difference in electrical potential measured across two points of the circuit. Potential difference is always less than EMF. The supply voltage is called potential difference. It is represented by V.

Voltage

It is the electric potential between two lines or phase and neutral. Its unit is volt. Voltmeter is used to measure voltage and it is connected parallel between the supply terminals.

Volt

It is defined as when a current of 1 ampere flows through a resistance of 1 ohm, it is said to have potential difference of 1 volt.

Current

It is the flow of electrons in any conductor is called current. It is represented by 'I' and its unit is Ampere. Ammeter is used to measure the current by connecting series with the circuit.

Ampere

When 6.24×10^{18} electrons flow in one second across any cross section of any conductor, the current in it is one ampere.(or) If the potential difference across the two ends of a conductor is 1 volt and the resistance of conductor is 1 ohm then the current through is 1 ampere.

Resistance

It is the property of a substance to oppose to the flow of electric current through it, is called resistance. Symbol: R, Unit : Ohm (Ω), Ohm meter is used to measure the resistance.

Ohm

If the potential difference across the two ends of conductor is 1 volt and the current through it is 1 ampere, then the resistance of the conductor is 1 Ohm.

Laws of resistance

The resistance offered by conductor depends on the following factors.

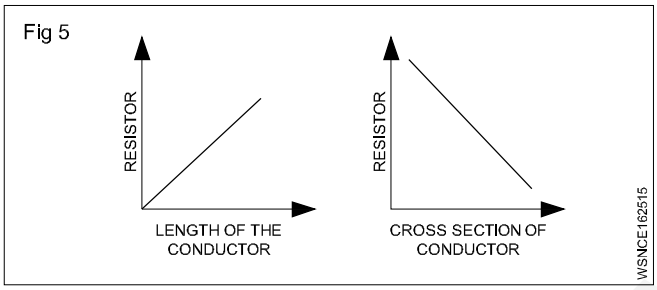
The resistance of the conductor

1 is directly proportional to the length of the conductor ($R \alpha L$)

2 Varies inversely proportional to its cross sectional area of the conductor $\left(R \alpha \frac{1}{A}\right)$

3 Depends on the material with which it is made.

Fig 5



4 depends on the temperature of the conductor

$$R \alpha L ; \quad R \alpha \frac{1}{A}; \quad R \alpha \frac{L}{A}; \quad R = \rho \frac{L}{A}$$

Specific resistance

The specific resistance of a material is the resistance offered to a current it passed between the opposite faces of the unit cube of the material. Specific resistance is measured in Ohm - m or micro ohm - cm.

Each material has its own specific resistance or resistivity.

E.g. : Copper - 1.72 $\mu\Omega$ cm, Silver - 1.64 $\mu\Omega$ cm, Eureka - 38.5 $\mu\Omega$ cm, Iron - 9.8 $\mu\Omega$ cm, Aluminium - 2.8 $\mu\Omega$ cm, Nickel - 7.8 $\mu\Omega$ cm.

$R = \frac{\rho l}{A}$ ohm cm

R = Resistance in ohms

l = Length of the conductor in cm

ρ = Specific Resistance in ohm cm (symbol pronounced as rho)

A = Area of cross - section in cm^2

Basic Electricity - Conductor, insulator, types of connections - series and parallel

Conductors

Some materials and metals readily allow passage for electric current to flow. In such materials, called conductors, electrons are able to pass readily from atom to atom.

Properties of conductors

A good conductor should have the following properties.

Electrical properties

- The conductivity must be good.
- Electrical energy spent in the form of heat must be low.
- Resistivity must be low (to reduce voltage drop and loss).
- Increase in resistance with temperature must be low.

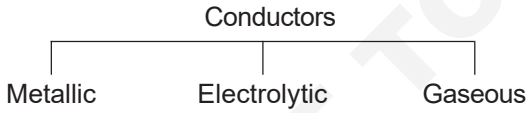
Mechanical properties

- Ductility (the property of being drawn into thin wires).
- Solderability: the joint should have minimum contact resistance.
- Resistance to corrosion: should not get rusted when used outdoors.
- Should withstand stress and strain.
- It should be easy to fabricate.

Economical factors

- Low cost.
- Easy availability.
- Easy to manufacture.

Classification of conductors



The best conductors are metallic. The commonly used conductors in electrical appliances and machines are described hereunder.

Silver

It is a soft and extremely malleable metal. Even though it is the best conductor, its use is limited because of its high cost.

Copper

It is a very good conductor. It is malleable and ductile, and also has high resistance to corrosion by liquids. Therefore, it is widely used for wires, cables, overhead conductors, bus bars and conducting parts of various electrical appliances.

Aluminium

It is a metal light in weight. It is also ductile, malleable and a good conductor of electricity. Nowadays, it is more widely used (since it is cheaper than copper) for wires and cables. All aluminium conductors (AAC) and aluminium conductors (steel reinforced) (ACSR) are used in overhead and transmission lines. (More details on copper and aluminium are furnished under the topic 'non-ferrous metals and alloys as applicable to electrical trades').

RESISTANCE WIRES

These are conductors with very high resistance for specific applications like filaments of incandescent lamps, heating elements etc. The following are a few examples:

1	Tungsten	2	Nichrome	3	Eureka
4	German silver	5	Manganin	6	Platinum
7	Mercury	8	Carbon	9	Brass.

The resistance values of the metallic resistances will increase with increase in temperature.

insulators

Description

These are the materials which offer very high resistance to the flow of current and make current flow very negligible or nil. These materials have very high resistance - usually of many megohms (1 megohm = 10^6 ohms) per centimetre cube. The insulators should also possesses high dielectric strength. This means that the insulating material should not break down or puncture even on application of a high voltage (or high electrical pressure) to a given thickness.

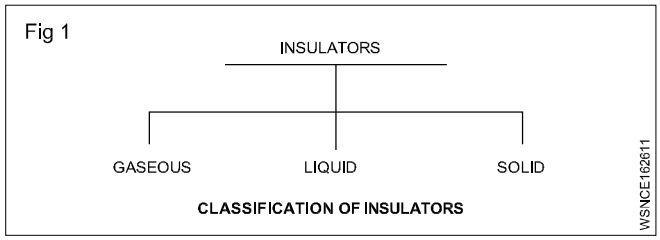
Properties of insulators

The main requirements of a good insulating material are:

- high specific resistance (many megohms/cm cube) to reduce the leakage currents to a negligible value
- good dielectric strength i.e. high value of breakdown voltage (expressed in kilovolts per mm)
- good mechanical strength, in tension or compression (It must resist the stresses set up during erection and under working conditions.)
- little deterioration with rise in temperature (The insulating properties should not change much with the rise in temperature i.e. when electrical machines are loaded.)
- non-absorption of moisture, when exposed to damp atmospheric condition. (The insulating properties, specially specific resistance and dielectric strength decrease considerably with the absorption of even a slight amount of moisture.)

Classification of insulators (Fig 1)

Fig 1



Air is an example of a gaseous insulator. Other examples are hydrogen, nitrogen and inert gases.

Liquid insulators

Mineral oils, synthetic liquids, resins and varnishes are the liquid insulators.

Transformer oil

In transformers the oil is used as an insulator and also for cooling of the transformer windings by convection. Therefore, the transformer oil should be dry and purified, since the presence of moisture will reduce the dielectric strength of the oil.

Purpose of transformer oil

- Transfer of heat by convection, from winding and core to the cooling surfaces.
- It maintains the insulation of winding and also extinguishes fire that occurs due to faults occurring in the windings.

Precaution

The insulating value of a transformer oil is reduced due to the formation of sludge as a result of oxidation due to air and temperature. To minimise oxidation, the oil should not be exposed to air.

Sludge is also formed due to the presence of acids and alkalis.

Sludge formation

- Reduces the rate of heat transfer.
- Blocks the ducts.
- Increases the operating temperature.

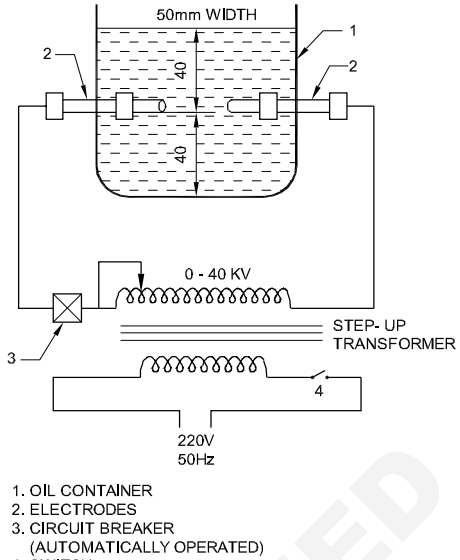
To prevent moisture from entering the oil, the whole apparatus is made airtight, and calcium chloride, silicagel fillets are used.

Testing of transformer oil as per ISI Standard (Fig 2)

Dielectrical strength test (Refer to Fig 2) : The oil should be 40 mm above and 40 mm below the electrodes. The gap between the two electrodes should be kept at 4 mm ± 0.02 mm).

A high voltage is applied across the electrodes through a step-up transformer, and increased till there is a spark in between the electrodes. The voltage noted on the voltmeters, when the spark occurs, is the breakdown voltage or dielectric strength of the oil. This is the maximum voltage the oil can withstand.

Fig 2



According to ISI specifications, the oil should be able to withstand 40 kV for one minute with a gap (4 mm ± 0.02 mm) between the electrodes and with the diameter of the electrodes as 13 mm.

Moisture test : In this test, an oil sample is cooled in a closed vessel down to 15-25°. A dry test tube, 12.5 mm in diameter and 125 mm long, is taken and an adequate quantity of oil is poured into it.

The tube containing the oil is heated rapidly with the help of an electric heater till the oil begins to boil. During the process, oil should not produce cracking.

The other tests are:

- acidity test
- sludge resistance test.

Electrical insulating varnishes

They are of two types

Oil and resin varnishes.

Solid insulators/insulating materials

Sl. No.	Classification	Examples
1	Mineral insulators	Mica, marble, slate.
2	Vitreous materials	Glass, quartz, procelain.
3	Rubber and rubber products	Rubber, vulcanised (India) rubber (V.I.R) ebonite
4	Waxes and compounds	Paraffin wax, bitumen.
5	Fibrous materials	Asbestos, paper, wood, Press pahn, leatheroid, cotton, silk, tapes etc.
6	Synthetic products	Bakelite, shellac, oil (for Transformer, Switchgear etc).

Paper

Various grades of insulating paper are available for use in capacitors, cables, etc. Paper, if moist, loses its insulating property. Therefore, it is used in an impregnated condition.

Wood

It is impregnated with oil or other substance for use as an insulator.

For example, in machine windings, bamboo wood is used as slot wedges.

Press board

It is widely used in windings to insulate parts which support windings. It is also used as spacers in electrical devices and transformers.

Asbestos

A fibrous, incombustible, fire- proof material - used for panel boards and as frames for winding resistance wires of regulators, rheostats etc.

Cotton

It is soaked in paraffin to avoid moisture. It is a good insulator for low voltages. It is used in conductors for armatures and field coils.

Silk

Like cotton, it is used for small jobs like telephone coils.

Tapes

Tapes of various types are used, such as cotton, silk, jute etc either pure or in impregnated form.

Empire cloth

It is made by varnishing a cotton cloth, silk or paper. It is not effected by moisture. It is available in yellow and black colours in different sizes. It is used as slots insulation in winding works and for coil insulation.

Press pahn

Press pahn is a form of paper made from hemp, rags, and wood pulp by special chemical treatment. It is widely used for lining armature slots, insulating coil sides, etc.

Leatheroid

It is a tough material made from cotton rags with chemical treatment. It is unaffected by grease or oil and is used for slot and coil insulation, transformer core coverings, etc.

Adhesive tape

It is used widely for taping of ends of conductors, leads and connections. Adhesive tape is made from cotton fabric coated with a compound of rubber, bitumen, resin, gum, etc. It dries when exposed to air. It is available in sizes $\frac{1"}{2}$, 3/4", 1" etc. These are also available as P.V.C. adhesive tape, cotton and bitumen tapes.

Bitumen

It is used for filling cable jointing boxes and for sealing the tops of the batteries etc. It is waterproof, but it will crack under certain conditions. It can be valcanised in the same manner as rubber.

Mica

It is a mineral and available as large slabs. It can be easily separated into thin sheets. It is fireproof, waterproof, and is a good insulator. It should be used carefully since it is liable to crack. It is used in heating elements of electric iron etc.

Marble and slate

Marble and slate are mechanically strong insulators and are non- hygroscopic. When polished they form good mountings for switchboards, switches, resistance frames, etc. Slate is used generally for low voltages.

Micanite

It is made by sticking together pieces of mica with insulating cement like shellac. It can be bent to any shape by heating and pressing. Therefore, it is used as insulator for slots of armatures and to insulate the commutator from the shaft.

Paraffin wax

It melts at 55°C and does not absorb water. It is used to impregnate paper, wood, pressboard etc to reduce their moisture absorption.

Bakelite

It can be moulded to any shape. It is heat-resistant and highly insulating. It will not absorb oil and moisture. It is used for bodies of switches, plugs, holders, regulators etc.

Rubber

It has high insulating properties. It is used mainly on lighting cables and for flexible cables. It deteriorates gradually when exposed to atmosphere. Rubber is being replaced now by elastic plastics such as PVC or polyethylene which can resist alkalis, acids and mineral oils.

Valcanised India Rubber (VIR)

This is manufactured by treating pure rubber with sulphur. It is stronger than pure rubber and is not affected much by change in temperatures. It is used as coverings for low and medium voltage wires and cables.

Ebonite or vulcanite

Ebonite or Vulcanite is vulcanised rubber containing about 30% to 50% of sulphur, and subjected to a prolonged heating at 150°C. The material is hard and can be moulded into different shapes. It is less affected by chemicals and moisture. It is used for making containers of lead acid batteries, cases for instruments and switchgears, terminal plates and low voltage panel boards etc. It should not be subjected to heat.

Shellac

It is a good varnish which is used to improve the insulation and moisture resisting properties of paper, cloth, wood, slate etc.

Enamel

By this, an insulation coating is given on winding wires.

Polychloroprene (PCP)

It is a plastic material used for insulation of cables. It is resistant to oil and petrol. It can be used in conditions of exposure to sulphur fumes, steam, ammonia, lactic acid and direct sunlight.

Glass

It is heat-resistant and suitable for high temperatures. It is used as insulators, envelopes for lamps, radio tubes etc.

Quartz

Quartz (Silica) is a good insulator. As it has a very low temperature coefficient of expansion, it does not crack with sudden variations in temperature. It is used for pyrometer sheaths, for heating elements, sparking plugs, etc.

Porcelain

Porcelain is not so brittle as glass and is very widely used for carrying bare conductors, for making fuse carriers and other electrical fittings.

Red fibre

Mainly used in motor and transformer winding work, for slot insulation, separators etc.

Insulators classified according to their temperature limits

The permissible temperature limit at which the insulators may be worked safely without deterioration, depends upon the type and class of the insulation as detailed below. (IS:1271/1958)

Class Y – maximum temperature 90°C

Cotton, silk, paper products, press board, wood, valcanised fibre - not impregnated or immersed in oil.

Class A – maximum temperature 105°C

Cotton, silk, paper products, wood, valcanised fibre when impregnated or immersed in liquid dielectric, varnished paper and wire enamel (class A).

Class E – maximum temperature 120°C

Wire enamel, cotton fabric and paper laminates treated with oil, modified asphalt and synthetic resins, varnished polyethylene, textile treated with suitable varnish.

Class B – Maximum temperature 130°C

Glass fibre, asbestos, varnished glass fibre, textile, varnished asbestos, built up mica treated with synthetic resin varnishes.

Class F – maximum temperature 155°C

Similar to class B materials but treated with silicone resins.

Class H – maximum temperature 180°C

Same as class F materials but treated with silicone resins of higher thermal stability than class F.

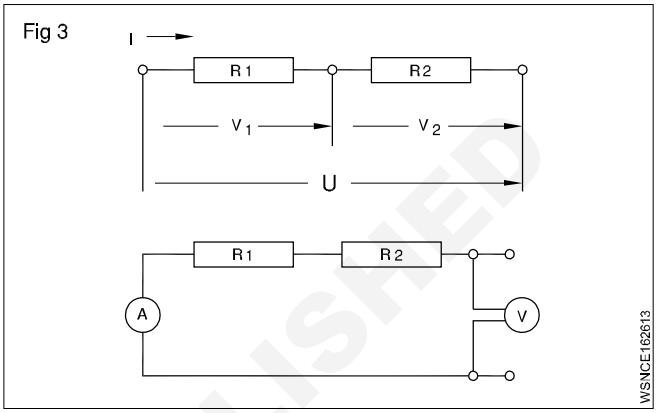
Class C – maximum temperature above 180°C

Mica, porcelain and other ceramics, glass, quartz, asbestos, treated glass fibre textile, treated asbestos, built up mica treated with silicone resins possessing superior thermal stability (limited stability up to 225°C).

Series Connection

The total resistance is equal to the sum of all the resistances. In a series connection the end of the first load is connected to the beginning of the second load and all loads are connected end to end. (Fig 3)

Fig 3



Features of series connection:

- The same current flows through all the loads.
- The voltage across each load is proportional to the resistance of the load.
- The sum of the voltages across each load is equal to the applied voltage.
- The Total resistance is equal to the sum of all the resistances.

$I = I_1 = I_2 = ...$

$V = V_1 + V_2 + ...$

$R = R_1 + R_2 + ...$

Example

Three resistances of 3 ohms, 9 ohms and 5 ohms are connected in series. Find their resultant resistance.

Solution

$R = R1 + R2 + R3$

$= 3\ \Omega + 9\ \Omega + 5\ \Omega$

Total resistance $= 17\ \Omega$

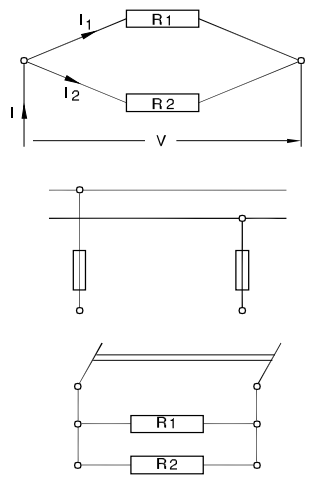
Parallel connection

In a parallel connection the beginning and the ends of the loads are connected together.

Features of parallel connection:

- The current flowing through each load depends upon the resistance of the load.
- The voltage across each load is the same and is equal to the voltage applied to the circuit.

Fig 4



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- The total resistance of a parallel connection is always smaller than the smallest resistance in the circuit.
- In parallel connection the reciprocal of the total resistance is equal to the sum of the reciprocals of all resistances in the circuit.

$I = I_1 + I_2 + \ldots$

$V = V_1 = V_2 \ldots$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots$$

Example

Two resistances of 4 ohms and 6 ohms are connected in parallel. Determine the total resistance.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \quad \text{(since parallel connection)}$$

Therefore $\frac{1}{R} = \frac{1}{4} + \frac{1}{6} = \frac{10}{24} = \frac{5}{12}$

Therefore $R = \frac{24}{10}$ ohms = 2.4 ohms

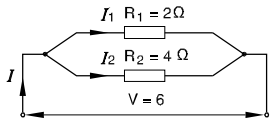
Example

Two resistors of 2 and 4 ohms are switched in parallel to a 6V battery

– Calculate the total resistance

– Find the total current and partial current.

Fig 5



RESISTORS IN PARALLEL

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Solution

Total resistance

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{1}{2} + \frac{1}{4} = \frac{2+1}{4}$$

$$= \frac{3}{4}\Omega$$

$$R_{tot} = \frac{4}{3} = 1\frac{1}{3}\Omega$$

I Total = $I_1 + I_2$ current

$$But\, I_1 = \frac{U}{R_1} = \frac{6V}{2\Omega} = 3A$$

$$I_2 = \frac{U}{R_2} = \frac{6V}{4\Omega} = 1.5A$$

I total = 3A + 1.5A

= 4.5 Amp

Basic Electricity - Ohm's law, relation between V.I.R & related problems

Ohm's law

V - Voltage in volts

I - Current in Ampere

R - Resistance in ohms.

In any closed circuit the basic parametres of electricity (Voltage, Current and resistance) are in a fixed relationship to each other.

Basic values

To clarify the basic electrical values, they can be compared to a water tap under pressure

Water pressure	- electron pressure	- Voltage
Amount of water	- electron flow	- Current
throttling of tap	- obstruction to electron flow	- Resistance

Fig 1

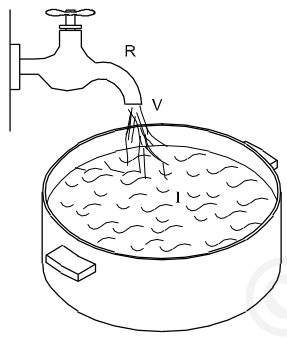

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Fig 2

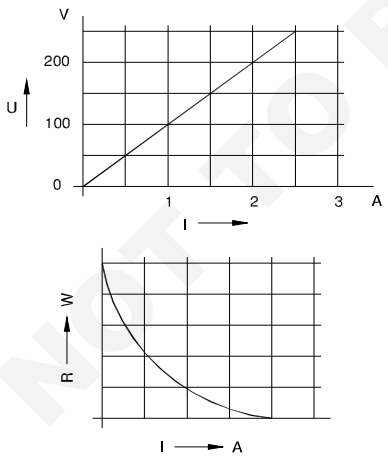

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Fig 3

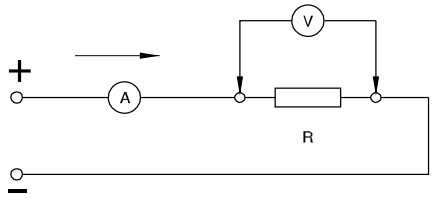

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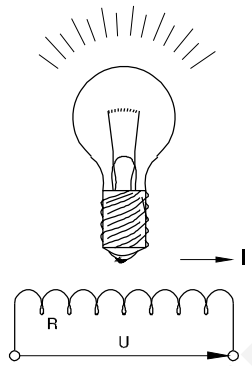
Fig 4



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Relationships

If the resistance is kept constant and the voltage is increased, the current is increased

$$I \propto V$$

If voltage is constant and the resistance is increased, current is decreased

$$I \propto \frac{1}{R}$$

Ohm's law

From the above two relationships we obtain Ohm's law,

$I = \frac{V}{R}$ which is conveniently written as V = R.I.

Ohm's law states that at constant temperature the current passing through a closed circuit is directly proportional to the potential difference, and inversely proportional to the resistance.

By Ohm's law $I = \frac{V}{R}$

EXAMPLE

A bulb takes a current of 0.2 amps at a voltage of 3.6 volts. Determine the resistance of the filament of the bulb to find R. Given that V = 3.6 V and *I* = 0.2 A.

To find 'R'. Given that V = 3.6V and I = 0.2 A

Therefore V = I x R

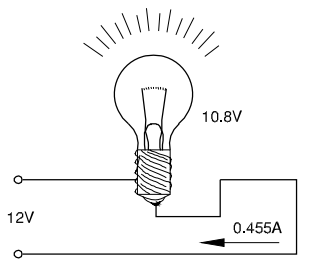
3.6 V = 0.2 A x R

Therefore $R = \frac{3.6V}{0.2A} = 18 ohms$

Example

The voltage supply to a filament lamp is 10.8V. The voltage should be 12V. Find out loss of voltage.(Fig 5)

Fig 5



Voltage drop = 12V – 10.8 = 1.2V

The supply voltage is called Potential difference.

Example

The Internal resistance of a dynamo is 0.1 ohm. The voltage of dynamo is 12V. What is the Voltage of dynamo when a current of 20 amps being supplied to an outside circuit.

Solution

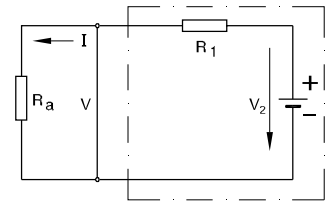
Voltage drop = Current x Internal resistance

= 20 x 0.1 volts

= 2 volts

Example (Fig 6)

Fig 6



The Internal resistance of a Battery is 2 ohms. When a resistance of 10 ohms is connected to a battery it draws 0.6 amps. What is the EMF of the battery.

P.D = Current flowing x Resistance

= 0.6 A x 10Ω

= 6 volts

V.D = Current flowing x Internal resistance of battery

= 0.6 x 2 volts

= 1.2 volts

EMF of the Battery = (6.00 + 1.2)V

= 7.2 volts

Resistance connections

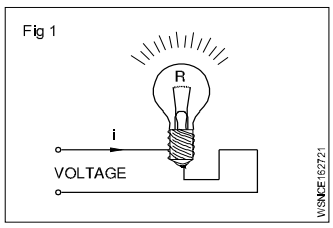
V - Voltage (in volts)

R - Resistance (in ohms)

I - Current intensity (in Amperes)

Assignment

1

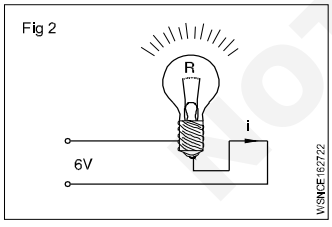


R = 40 Ohms

I = 6.5 Amps

V = _____ Volts

2

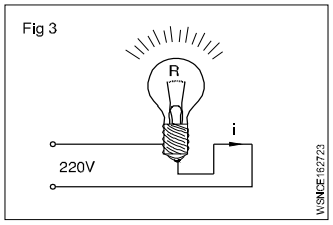


V = 6 Volts

I = 0.5 Amps

R = _____ Ohms

3

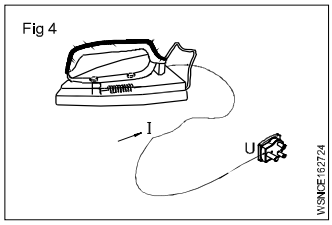


V = 220 Volts

R = 820 Ohms

I = _____ Amps

4

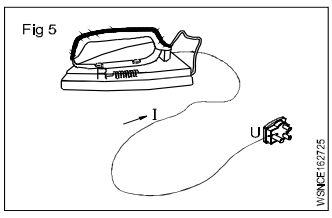


I = 4.5 Amps

V = 220 Volts

R = _____ Ohms

5

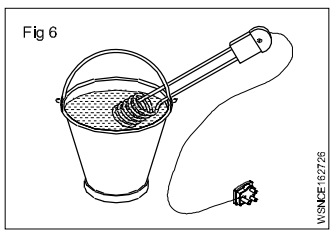


R = 50 Ohms

V = 220 Volts

I = _____ Amps

6



V = 110 Volts

I = 4.55 Amps

R = _____ Ohms

Basic Electricity - Electrical power, energy and their units, calculation with assignments

Electric Power

In mechanical terms we defined power as the rate of doing work. The unit of power is Watt. In an electrical circuit also the unit of electrical power is 1 Watt. In mechanical terms 1 Watt is the work done by a force of 1 N to move the body through 1 metre in one second. In an electrical circuit, the electromotive force overcomes the resistance and does work. The rate of doing work depends upon the current flowing in the circuit in amperes. When an e.m.f of one volt causes a current of 1 ampere to flow the power is 1 Watt.

Hence Power = Voltage x Current

$$P = V \times I$$

Power in Watts = Voltage in Volts x Current in Amperes

Electric work, energy

Electrical work or energy is the product of electrical power and time

Work in Watt seconds = Power in Watts x time in sec

$$W = P \times t$$

Since 1 joule represents 1 Watt x 1 sec, which is very small, larger units such as 1 Watt hour and 1 kilowatt hour are used.

1 W.h = 3600 Watt sec.

1 Kwh = 1000 Wh = 3600000 Watt sec

FPS System

1 HP = 746 watts

= 0.746 K.W

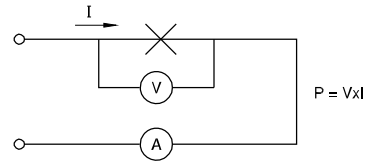
Metric System

1 HP = 735.6 watts

= 0.7356 K.W

Note: The charge for electric consumption is the energy cost per Kwh and it varies according to the country and states.

Fig 1



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V - Voltage (Volts) V

i - Current Intensity (Amperes) A

P - Power (Watts, Kilowatts) W, kW

W - Work, Energy (Watt hour, Kilowatt hour) wh, Kwh

t - time (hours) h

Fig 2

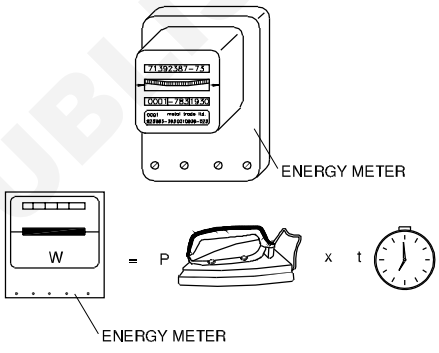

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Table of analogies between mechanical and electrical quantities

Mechanical quantity	Unit	Electrical quantity	Unit
Force 'F'	N	Voltage 'V'	V
Velocity $v = \frac{\text{Displacement}}{\text{Time}}$	m/s	Current *I*	A
Time *t*	seconds	Time *t*	seconds
Power $P = F \times v$	N $\frac{m}{sec}$	Power $P = V \times i$	$W = V \times A$
Energy = $F \times v \times t$	j = Nm	Energy $W = V \times i \times t$	$j = W \times s$

$$W = V I = I^2R = \frac{V^2}{R}$$

$$R = \frac{V}{I} = \frac{V^2}{W} = \frac{W}{I^2}$$

$$V = IR = \frac{W}{I} = \sqrt{WR}$$

$$I = \frac{V}{R} = \frac{W}{V} = \sqrt{\frac{W}{R}}$$

Example

1 Calculate the power rating of the lamp in the circuit, if 0.25 amperes of current flows and the voltage is 240 volts.

P = V x I

V = 240 Volts

I = 0.25 Ampere

Therefore Power = 240 Volts x 0.25 Amperes

= 60 Volts Ampere

But 1 Watt = 1 Volt x 1 Ampere

Therefore Power = 60 Watts

2 A current of 15 amperes flow through a resistance of 10 Ohms. Calculate the power in kilowatts consumed.

Given that R = 10 and I = 15A

Power = V x I = I x R x I = I^2 x R

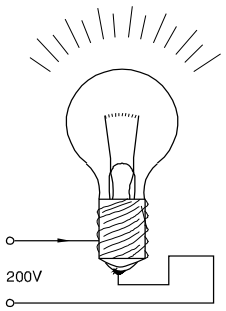
Therefore Power = 15^2 x 10 = 2250 Watts = 2.25 kW

3 At a line voltage of 200 Volts a bulb consumes a current of 0.91 amperes. If the bulb is on for 12 hour calculate the work in Wh to find the work given that V = 200 Volts.

I = 0.91 Amps.

t = 12 hours

Fig 3



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Therefore Power=V x I = 200 Volts x 0.91 Amps

= 182 Watts

Therefore Work = P x t = 182 Watts x 12 hours

= 2184 Watt hour.

4 An adjustable resistor bears the following label: 1.5 k Ohms/0.08 A. What is its rated power?

Given: R = 1.5 k Ohms; I = 0.08 A

Find: P

V = R x I = 1500 Ohms.0.08 A = 120 volts

P = V x I = 120 volts.0.08 A = 9.6 W alternatively: P = 1^2.R = $(0.08\ A)^2$.1500 Ohms = 9.6 W.

5 Find the current and power consumed by an electric iron having 110Ω resistance when feed from a 220 v supply

Resistance of electric iron (R) = 110 ohms

Voltage (V) = 220 volts

Current (I) = $\frac{V}{R}$

= $\frac{220}{110}$ 2 amperes

Power (w) = V x I

= 220 x 2

= 440 watts

6 Find the total power if four 1000 W, 180 volt heaters are connected in series across 240 V supply and current carrying capacity is 15 amp. Find the total power.

Connection = Series

No. of heaters = 4

Heater power (W) = 1000 watts

Heater voltage = 180 V

Supply voltage = 240 V

Heater resistance (R) = $\frac{V^2}{W}$

= $\frac{180 \times 180}{1000} = \frac{324}{10}$

= 32.4 ohms

Total resistance = 32.4 x 4 = 129.6 ohms

Total current (I) = $\frac{V}{R}$

= $\frac{240}{129.6}$ = 1.85 amperes

Total Power (W) = V x I

= 240 x 1.85 = 444 watts

7 If a 40 watt fluorescent lamp draws a current of 0.10 ampere. How much voltage will be required to illuminate it?

Lamp power (W) = 40 watt

Current (I) = 0.10 ampere

Voltage (V) = $\frac{W}{I}$

= $\frac{40}{0.1}$ = 400 volts

8 Find the cost of running 15 HP motor for 15 days @ 6 hrs per day and the cost of energy is Rs. 3 per unit.

Motor power (w) = 15 HP

= 15 x 746 = 11,190 watts

Consumption per day = 11,190 x 6

= 67140 = 67.14 KWH

Consumption for 15 days = 67.14 x 15

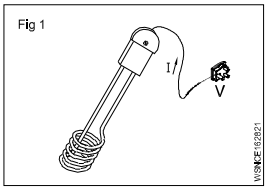
= 1007 KWH (or) unit

Cost per unit = Rs. 3

Cost for total energy = 3 x 1007 = Rs. 3021

Assignment

1



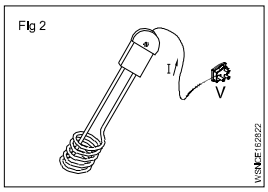
Current Consumed

I = 0.136 A

Voltage 'V' = 220 V

P = _____ Watts

2

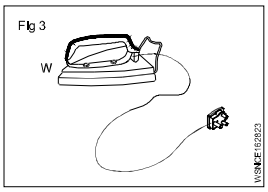


P = 500 Watts

I = 2.27 A

V = _____ v

3

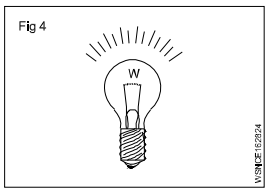


P = 750 W

V = 220 v

I = _____ A

4

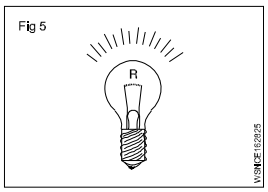


P = 60 W

V = 200 v

R = _____ W

5

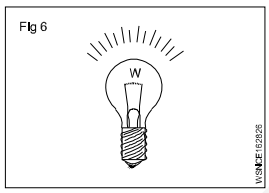


I = 0.455 A

R = 484 ohms

P = _____ ohms

6

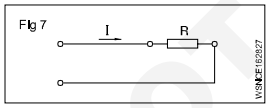


P = 550 W

R = 22 ohms

I = _____ A

7

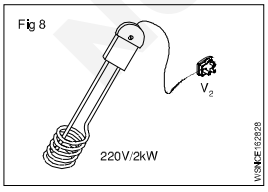


P consumed = 1.8 kW

R = 8 ohms

V = _____ v

8



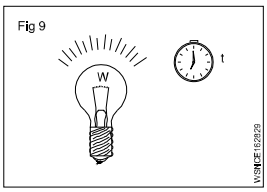
I consumed = ____ A

P = 2 kW

V_1 = 220V (Heating element voltage)

R = _____

9



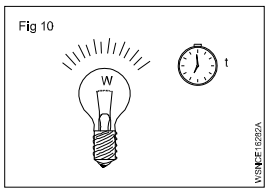
P = 100 W

t = 1 hour

Energy consumption

= _____ kWh

10



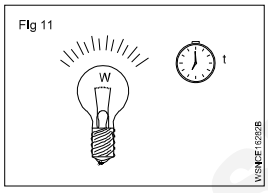
Energy consumed

'W' = 1 kWh

Power 'P' = 100 W

t = _____ hr

11

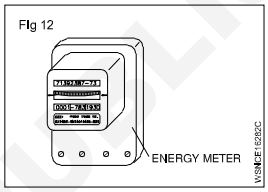


W = 1.5 kWh

t = 45 min.

P = _____ kW.

12



Energy metre reading

W_1 = 6755.3 kWh

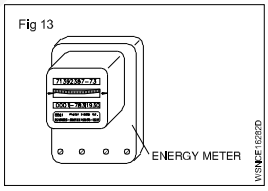
Increases to W_2

= 6759.8 kWh

t = 45 min.

P = _____ kW.

13



Power consumed

'P' = 6.2 kW

t = 8 hours

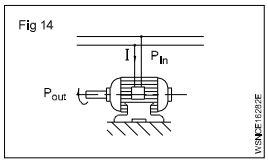
Charge per kwh

= 1.25 Rupees

Total cost

= _____ Rupees

14



I = 5.45 A

V = 220 v

Energy consumed

= 1 kWh

t = _____ hr.

Basic Electricity - Electrical power, HP, energy and units of electrical energy

Electric Power

In mechanical terms we defined power as the rate of doing work. The unit of power is Watt. In an electrical circuit also the unit of electrical power is 1 Watt. In mechanical terms 1 Watt is the work done by a force of 1 N to move the body through 1 metre in one second. In an electrical circuit, the electromotive force overcomes the resistance and does work. The rate of doing work depends upon the current flowing in the circuit in amperes. When an e.m.f of one volt causes a current of 1 ampere to flow the power is 1 Watt.

The rate of work is called Power. Its units is watt. In a circuit with one volt pressure the quanity of electric current produced represents one watt of power. Power is obtained from circuit voltage multiplied by current.

If voltage is V and current I, then power

$$P = V \times I \text{ watts}$$

According to Ohm's law

$$V = I \times R$$

$$P = I \times R \times I$$

$$= I^2 \times R \text{ watts}$$

Horsepower Definition Science

Horsepower is defined as the work done per unit time and this term was invented by James Watt. It is a unit of power that relates a machine's power to the muscle power of a horse. It was once used to compare the output of steam engines to the power of draught horses. It was later expanded to include other piston engines, electric motors, turbines, and other machinery.

Horsepower

The practical unit of power is Horsepower.

one horsepower = 735.6 watts

FPS System	Metric System
1 HP = 746 watts	1 HP = 735.6 watts
= 0.746 K.W	= 0.7356 K.W

Energy

The capacity to work is called energy. In electricity energy is produced by power through work in a given time. In a circuit, power W is spent in time (t) then it will be said that energy used in circuit = W x t. The unit of energy is watt seconds or joule.

$\therefore$ Energy = Power in watt x time in seconds

Unit of Electrical Energy

This is bigger unit of energy. Its full name is Board of Trade Unit but in short it is called Unit. In this, power is measured in kW and the time is in hours. Its unit is kilowatt hour or kWh.

1 kWh = 1000 watt hours

= 1 unit of electricity

Example

1 A D.C motor takes 7.106 amps at 110 volts and has an efficiency of 80%. How much HP does it deliver?

Current (I) = 7.106 amp

volt (V) = 110 volts

efficiency = 80%

Horsepower = Output ?

Input = V x I watts

Power Input = 110 x 7.106 = 781.66 watts

$$\eta = \frac{\text{Output}}{\text{Input}} \times 100\%$$

$$80 = \frac{\text{Output}}{781.66} \times 100$$

$$\text{Power Output} = \frac{80 \times 781.66}{100} = 625.3 \text{ watts}$$

1 HP = 746 watts

$$\text{HP} = \frac{625.3}{746} = 0.8382$$

Motor power = 0.8382 HP

2 A factory having 4 motors each 5 HP runs 8 hours per day for 30 days. Calculate the electricity expenditure if Rs.2 per unit.

Motor = 5 HP

No. of Motor = 4

Runs per day = 8 hours

No.of days run = 30 days

Total HP = 4 x 5 = 20 HP

Total running = 8 x 30 = 240 hours

1 HP = 746 watts

1 kW = 1000 watts

$$\text{Total kWh} = \frac{20 \times 240 \times 746}{1000}$$

= 3580.8 kWh

1 kWh electricity charge = Rs.2

Total charge = 2 x 3580.8

Total electricity expenditure = Rs. 7161.60/-

Mensuration - Area and perimeter of square, rectangle and parallelogram

In Engineering field, an Engineer has to estimate the material, manpower, machinery, etc. required to prepare the geometrical objects. Hence we must be very conversant with all relevant formulae connected with geometrical objects.

Length - l unit

Breadth or width - b unit

Diagonal - d unit

Diameter - d unit

Radius - r unit

Semi perimeter - S unit

Perimeter - P unit

Circumference - C unit

Area - A unit^2

Total surface area - T.S.A unit^2

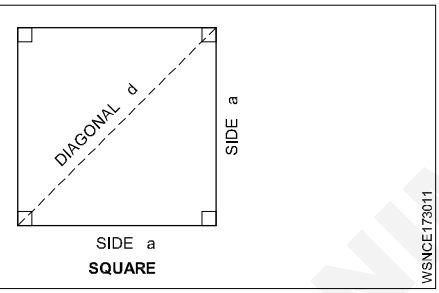
Lateral surface area - L.S.A unit^2

Volume - V unit^3

Square

This is also a four sided figure, opposite sides are parallel. All the four sides are equal. Angle between adjustment side are 90º.

Fig 1



$A = a^2$ (or) unit^2

$P = 4a$ unit

$d = \sqrt{2}\, a$ unit

$a = \frac{d}{\sqrt{2}}$ unit where $\sqrt{2} = 1.414$

Find the area of a brass sheet in the form of a square whose perimeter is 31.2 cm.

Perimeter(P) = 4a = 31.2 cm

$\therefore a = \frac{31.2}{4} = 7.8$ cm

Area (A) $= a^2$

$= 7.8 \times 7.8 = 60.84$ cm^2

Examples

1 Find out the circumference, diagonal and area of a square, whose side is 18 cm.

Side of the square (a) = 18 cm

Perimeter (P) = 4a

= 4 x 18 = 72 cm

Diagonal (d) = $\sqrt{2}$ x a

= $\sqrt{2}$ x 18 = 1.414 x 18

= 25.45 cm

Area (A) = a^2

= 18 x 18 = 324 cm^2

Perimeter of square = 72 cm

Diagonal = 25.45 cm ; Area = 324 cm^2

2 If the diagonal of a square measure 10 cm. Find area of the square.

Diagonal of the square (d) = $\sqrt{2}$ a = 10 cm

Side (a) = $\frac{d}{\sqrt{2}}$

Area (a^2) = $\frac{d}{\sqrt{2}} \times \frac{d}{\sqrt{2}} = \frac{d^2}{2}$

$= \frac{10^2}{2} = \frac{100}{2}$

= 50 cm^2

Area of the square = 50 cm^2

3 The perimeter of one square is 748 cm and that of Another is 336 cm. Find the total area of the two squares.

Side of the square (a) = $\frac{\text{Perimeter}}{4}$

1st square

Side (a) = $\frac{\text{Perimeter of 1st square}}{4}$

$= \frac{748}{4} = 187$ cm

Area (A) $= a^2$

= 187 x 187

= 34,969 cm^2

2nd square

Side (a) = $\frac{\text{Perimeter of 2nd square}}{4}$

$= \frac{336}{4} = 84$ cm

Area (A) $= a^2$

$= 84 \times 84$

$= 7{,}056\ cm^2$

Total area of two squares = 34,969 + 7,056

$= 42{,}025\ cm^2$

Assignment

1 Find the Area, Perimeter and diagonal of a square steel plate whose side measures 28.1 cm.

2 Find the area of a square whose diagonal is equal to 8.5 cm.

3 Find the area of the square if the side of the square is 28 cm.

4 Find its side if the area of the square field is 169 m^2.

5 Find the area of the square if the diagonal of the square is 20 cm.

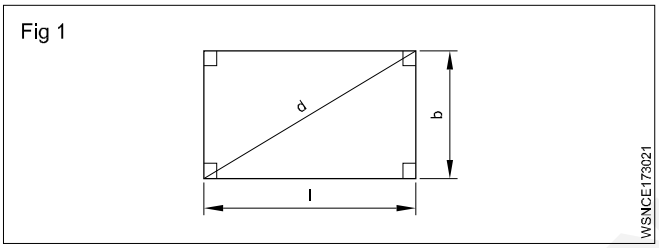
6 Find the perimeter of a square whose diagonal is 144 m.

7 Find the area if the perimeter of a square plot is 48 m.

Rectangle

This is a four sided figure. Opposite sides are parallel. Angles between adjacent sides are 90°.

Fig 1



A = Area = length x breadth = l.b.unit^2

P = Perimeter = 2 (l + b) unit

Diagonal = $\sqrt{l^2 + b^2}$ unit

Examples

1 Find the Area, Perimeter and diagonal of a rectangle whose length and breadth are 144 mm and 60 mm respectively.

Area = A $= l \times b\ unit^2$

$= 144 \times 60 = 8640\ mm^2$

Perimeter = P $= 2 (l + b)$ unit

$= 2(144 + 60)$

$= 2 \times 204 = 408$ mm

Diagonal = d $= \sqrt{l^2 + b^2}$ unit

$= \sqrt{144^2 + 60^2}$

$= \sqrt{20736 + 3600}$

$= \sqrt{24336} = 156$ mm

2 The perimeter of a rectangle is equal to 42 cm. If its breadth is 9 cm. Find the length of the rectangle.

Perimeter = 42 cm

Breath = 9 cm

Length = ?

Perimeter = P = 2(l + b)

$2(l + 9) = 42$

$l + 9 = 42 \div 2$

$l + 9 = 21$

$l = 21 - 9$

l = 12 cm

3 The perimeter of a rectangle is 48 cm and its length is 4 cm more than its width. Find the length and breadth of the rectangle.

Perimeter (P) = 48 cm

Breath (b) $= x$

Length (l) $= x + 4$

$2(l + b)$ = Perimeter

$2(x + 4 + x) = 48$

$2(2x + 4) = 48$

$4x + 8 = 48$

$4x = 48 - 8$

$x = \frac{40}{4} = 10$

x = breadth = 10 cm

length $= x + 4 = 10 + 4$ **= 14 cm**

4 How many rectangular pieces of 50 cm x 20 cm can be cut out from a sheet of 1000 cm x 500 cm.

Sheet size = 1000 cm x 500 cm

Size of the rectangular piece to be cut = 50 cm x 20 cm

No. of pieces to be cut in lengthwise = $\frac{1000}{50}$ = 20

No. of pieces to be cut in breadthwise = $\frac{500}{20}$ = 25

Total no. of pieces to be cut out = 20 x 25

= 500

5 The perimeter of a rectangle is 320 metre. Its sides are in the ratio of 5:3. Find the area of the rectangle.

Ratio = 5:3 = l : b

length l = $5x$

breadth b = $3x$

2(l + b) = Perimeter

$2(5x + 3x)$ = 320

$2(8x)$ = 320

$16x$ = 320

$x = \frac{320}{16} = 20$

$l = 5x$ = 5 x 20 = 100 m

$b = 3x$ = 3 x 20 = 60 m

Area = l x b (length = 100m, breath = 60m)

= 100 x 60

Area = 6000 m^2

Assignment

1 Find the area of a rectangular plot whose sides are 24 metres and 20 metres respectively. Also find the perimeter of the plot.

2 How many rectangular pieces of 5 cm x 4 cm will you get out of 65 cm x 30 cm brass sheet?

3 Find its breadth and area if the perimeter of a rectangle is 400 metre and its length is 140 m. .

4 Find its area, if the opposite sides of a rectangle are 64 cm and 25 cm respectively.

5 What is the width of the rectangle if a rectangle has an area of 224 cm^2 and length 16 cm.

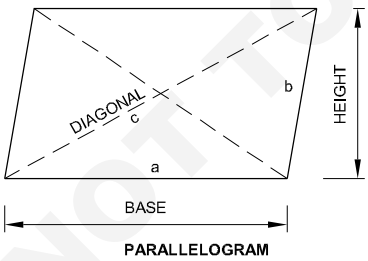
6 What is the length of the diagonal of a rectangle with sides 16 cm and 12 cm?

7 Find the area of the rectangle if the perimeter of the rectangle is 100 cm and the ratio of its length and breadth is 3:2.

Parallelogram

This is also a four sided figure, opposite side being parallel to each other.

Fig 1



Area of parallelogram = base x height

or = $2x\sqrt{s(s-a)(s-b)(s-c)}$

Where

$s = \frac{a+b+c}{2}$

a and b are adjacent sides.

P = 2(a+b)

Examples

1 The base and height of a parallelogram are 7.1 cm and 2.85 cm. Calculate its area.

A = base x height units^2

= 7.1 x 2.85

= 20.235 cm^2

2 Find the height of a parallelogram whose area is 20 cm^2 and base is 10 cm.

A = base x height units^2

$h = \frac{area}{base}$

$= \frac{20}{10}$

= 2 cm

3. Two sides of a parallelogram are 12 cm and 8 cm. The diagonal is 10 cm long. Find the area of the parallelogram.

Area A $= 2x\sqrt{s(s-a)(s-b)(s-c)}$ units^2

$s = \frac{a+b+c}{2}$

$= \frac{12+8+10}{2}$

$= \frac{30}{2}$

$= 15$

A $= 2x\sqrt{15(15-12)(15-8)(15-10)}$

$= 2x\sqrt{15 \times 3 \times 7 \times 5}$

$= 2x\sqrt{1575}$

$= 2 \times 39.686$

Area A $= 79.37$ cm^2

Assignment

1 Find the area of a parallelogram, if its base and height are 8.1 cm and 30.8 cm respectively.

2 Find the area of a parallelogram, if the sides of a field in the shape of parallelogram are 12 m and 17 m and one of the diagonal is 25 m.

3 Find the base of a parallelogram whose height is 12 cm and area is 120 cm^2.

4 Find the height of a parallelogram whose base is 40 cm and area is 320 cm^2.

5 Find the area of the land if the sides of a land in the shape of a parallelogram are 24 m and 28 m respectively and one of the diagonal is 30 m.

6 What is the perimeter of parallelogram if base is 10 cm and other side is 5 cm?

7 Find the area of parallelogram if its base and height are 25 cm and 12 cm.

8 Find the base of a parallelogram if height is 15 cm and area is 150 cm^2.

9 Find the height of a parallelogram if base is 80 cm and area is 640 cm^2.

10 Find the area of parallelogram if its base and height are 15 cm and 8 cm.

11 Calculate the perimeter and area of parallelogram if base, height are 12.7 cm, 5.5 cm and other side is 6.5 cm.

12 Find the height of parallelogram if the area is 20 cm^2 and base is 10 cm.

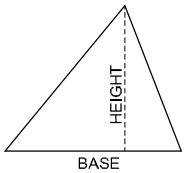
Mensuration - Area and perimeter of triangles

Triangles

Tri means three. Hence tri- angle means three angled figure. For construction of three angled figure, there should be three sides. Hence triangle means three sided figure. Sum of the three angles of any triangle = 180˚.

i Any triangle.

Fig 1



TRIANGLE

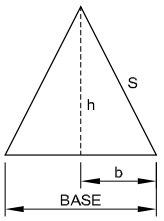
WSNCE173111

Area of any triangle = $\frac{1}{2}$ x Base x Height unit^2

ii Isosceles Triangle

In this triangle two of its sides are equal.

Fig 2



ISOSCELES TRIANGLE

WSNCE173112

Area of isosceles triangle = $\frac{1}{2}$ x Base x Height

Where

base = 2.b

s = One of equal sides (or) Slant height

h = Height = $\sqrt{s^2 - b^2}$

Area of isosceles triangle = $\frac{1}{2}$ x 2b x $\sqrt{s^2 - b^2}$

= b . $\sqrt{s^2 - b^2}$ unit^2

(Where b= half of base)

(or) Area of Isosceles triangle = $\frac{1}{4} b \sqrt{4a^2 - b^2}$ unit^2

a = Equal sides

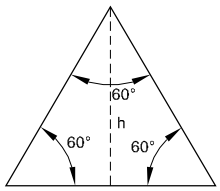
b = Base

iii Equilateral triangle

In this triangle all the three sides are equal. Hence angle between adjacent sides is 60˚ (Three angles total = 180˚)

angle between sides = $\frac{180}{3} = 60^\circ$

Fig 3



EQUILATERAL TRIANGLE

WSNCE173113

Area of equilateral triangle = $\frac{\sqrt{3}}{4}$ x side^2

= $\frac{\sqrt{3}}{4}$ x a^2 unit^2

Where $\sqrt{3}$ = 1.732

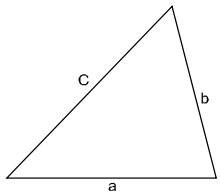
Perimeter P = 3a unit

P = $\frac{\sqrt{3}}{2}$a unit

iv Scalene triangle

In this triangle the sides are not equal. Angles between the sides, are also not equal. we may also call this triangle as irregular triangle.

Fig 4



SCALENE TRIANGLE

WSNCE173114

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ unit^2

where

a,b,c are sides of triangle

s = Semi perimeter = $\frac{a+b+c}{2}$ unit

v Right angled triangle

In this triangle, angle between one of two adjacent sides is 90^0. Right angle means 90°. That's why right angled triangle means, one of the angles of this triangle is definitely 90°.

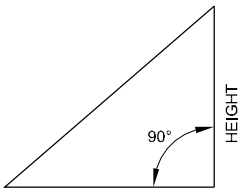
Area of right angled triangle

= $\frac{1}{2}$ x Base x Height

= $\frac{1}{2}$ bh unit^2

Hypotenuse = $\sqrt{\text{Base}^2 + \text{Height}^2}$

Fig 5



RIGHT ANGLED TRIANGLE

WSNCE173115

Where hypotenuse means, the diagonal or largest length of the side of right angled triangle.

Examples

1 Calculate its area if the base and height of a Right angled triangle are 10 cm and 3.5 cm respectively.

Base (b) = 10 cm

Height (h) = 3.5 cm

Area (A) = ?

$A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 10 \times 3.5$

= 17.5 cm²

2 Calculate the base of a triangle having an area of 15 cm² and height is 3.5 cm.

Area (A) = 15 cm²

Height (h) = 3.5 cm

Base (b) = ?

$\frac{1}{2} \times b \times h = A$

$\frac{1}{2} \times b \times 3.5 = 15$

$\mathbf{b} = \frac{15 \times 2}{3.5}$

= 8.57 cm

3 Calculate the height of a triangle whose area is 60 cm² and base is 10 cm.

Area (A) = 60 cm²

Base (b) = 10 cm

Height (h) = ?

$\frac{1}{2} \times b \times h = A$

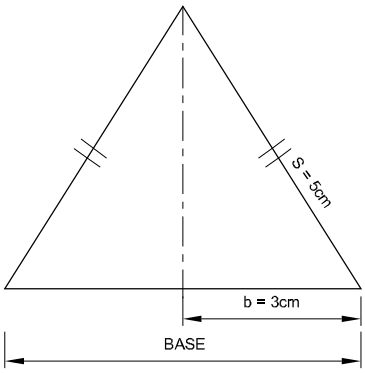
$\frac{1}{2} \times 10 \times h = 60$

$h = \frac{60 \times 2}{10}$

height h = 12 cm

4 Find the area of an isosceles triangle whose base is 6 cm long and each of the other two sides 5 cm long.

Fig 6



ISOSCELES TRIANGLE

WSNCE173116

Base (b) = 6 cm = $\frac{6}{2}$ = 3 cm

Equal sides or slant height 's' = 5 cm

Area (A) = ?

$A = b \times \sqrt{s^2 - b^2}$

$= 3 \times \sqrt{5^2 - 3^2}$

$= 3 \times \sqrt{25 - 9}$

$= 3 \times \sqrt{16}$

$= 3 \times 4$

$= 12$ cm²

or

$A = \frac{1}{4} b \sqrt{4a^2 - b^2}$

$= \frac{1}{4} \times 6 \sqrt{4x5^2 - 6^2}$

$= \frac{1}{4} \times 6 \times 8$

= 12 cm²

5 Find its height if an isosceles triangle has base of 200 mm and its area is 2000 mm².

Base = 200 mm

Area = 2000 mm²

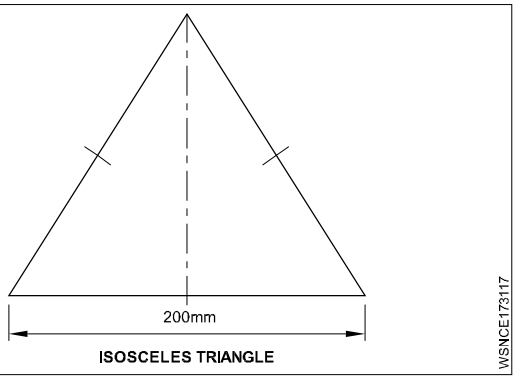
h = ?

$\frac{1}{2} \times b \times h = A$

$\frac{1}{2} \times 200 \times h = 2000$

$h = \frac{2000 \times 2}{200}$ **= 20 mm**

Fig 7



ISOSCELES TRIANGLE

6 Find the area of an equilateral triangle whose side is 5 cm.

$$\text{Area} = \frac{\sqrt{3}}{4} a^2 \text{ unit}^2$$

$$= \frac{1.732}{4} \times 5 \times 5$$

= 10.825 cm²

7 Calculate its perimeter if one side of an equilateral triangle is 55 mm long.

Side = 55 mm

Perimeter (P) = ?

P = 3a unit

= 3 x 55

= 165 mm

8 Find the area of the triangle having its sides are 9cm, 10cm and 12 cm.

$$\text{Semi Perimeter} = \frac{a+b+c}{2} \text{ unit}$$

$$= \frac{9+10+12}{2} = \frac{31}{2}$$

= 15.5 cm

$$\text{Area A} = \sqrt{s(s-a)(s-b)(s-c)} \text{ unit}^2$$

$$= \sqrt{15.5(15.5-9)(15.5-10)(15.5-12)}$$

$$= \sqrt{15.5 \times 6.5 \times 5.5 \times 3.5}$$

$$= \sqrt{1939.4375}$$

= 44.03 cm²

9 Find the cost of polishing on both sides of a triangular metal plate has sides 60 cm, 50 cm and 20 cm at the rate of Rs.1.35 per 100 cm²

$$\text{Semi Perimeter} = \frac{a+b+c}{2} \text{ unit}$$

$$= \frac{60+50+20}{2} = \frac{130}{2}$$

= 65 cm

$$\text{Area A} = \sqrt{s(s-a)(s-b)(s-c)} \text{ unit}^2$$

$$= \sqrt{65(65-60)(65-50)(65-20)}$$

$$= \sqrt{65 \times 5 \times 15 \times 45}$$

= 468.4 cm²

Area of polish on both sides = 2 x 468.4

= 936.8 cm²

Cost of polish per 100 cm² = Rs. 1.35

∴ Cost of polish is 936.8 cm² $= \frac{936.8}{100} \times 1.35$

= Rs. 12.65

10 Find the area of the right angled triangle with base 20 cm and height 8 cm.

Base b = 20 cm

Equal sides or slant height = 8 cm

Area (A) = ?

$$\text{Area (A)} = \frac{1}{2} \text{ x base x height unit}^2$$

$$= \frac{1}{2} \times 20 \times 8$$

= 80 cm²

11 Find the area of the right angled triangle if the sides containing the right angle being 10.5 cm and 8.2 cm.

$$\text{Area (A)} = \frac{1}{2} \text{ x base x height unit}^2$$

$$= \frac{1}{2} \times 10.5 \times 8.2$$

= 43.05 cm²

12 Calculate the perpendicular height of the triangle if the area of the right angled triangle is 19.44 m² and its one of the adjacent side containing the right angle being 5.4 m.

$$\frac{1}{2} \text{ x base x height unit}^2 = \text{Area}$$

$$\frac{1}{2} \times 5.4 \times h = 19.44$$

$$h = \frac{19.44 \times 2}{5.4}$$

= 7.2 m

13 Calculate the base of a right angled triangle having an area of 12.5 cm^2. If its height is 2.5 cm.

$$\frac{1}{2} \times \text{base} \times \text{height unit}^2 = \text{Area}$$

$$\frac{1}{2} \times b \times 2.5 = 12.5$$

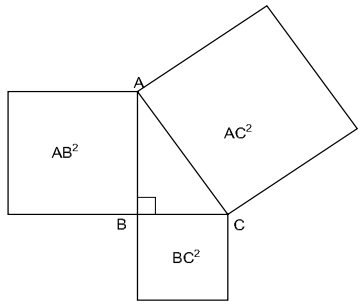
$$b = \frac{12.5 \times 2}{2.5}$$

= 10 cm

Pythagoras theorem

In a right angled triangle the area of the square drawn with the hypotenuse as the side is equal to the sum of the areas of the squares drawn with the other two sides.

Fig 8



$\angle B$ = 90°

AC = Hypotenuse

AB & BC = Adjacent sides

As per pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\therefore$ $AC = \sqrt{AB^2 + BC^2}$

1 Calculate the hypotenuse of a right angled triangle whose base is 5 cm and height is 12 cm.

As per pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= 12^2 + 5^2$

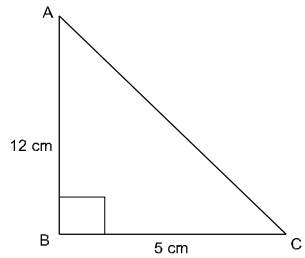
$= 144 + 25$

$= 169$

$AC = \sqrt{169}$

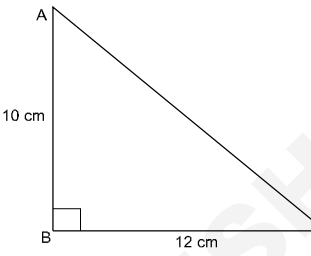
= 13 cm

Fig 9



2 What is the length of the hypotenuse of a right angled triangle, when the sides containing the right angles are 10 cm and 12 cm.

Fig 10



As per pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= 10^2 + 12^2$

$= 100 + 144$

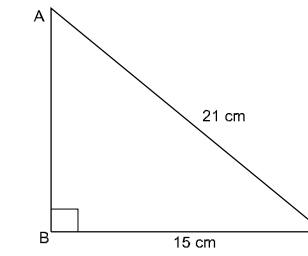
$= 244$

$AC = \sqrt{244}$

= 15.62 cm

3 Find the height of a right angled triangle whose base is 15 cm and hypotenuse is 21 cm.

Fig 11



As per pythagoras theorem,

$AB^2 + BC^2 = AC^2$

$AB^2 + 15^2 = 21^2$

$AB^2 = 441 - 225$

$= 216$

$AB = \sqrt{216}$

= 14.7 cm

Mensuration - Area and perimeter of circle, semi-circle, circular ring, sector of circle, hexagon and ellipse

Circle

It is the path of a point which is always equal from its centre is called a circle.

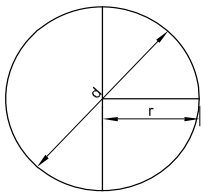
r = radius of the circle

d = diameter of the circle

$$\pi = \frac{22}{7} = 3.14$$

Area of the circle = πr^2

Fig 1



PROPERTIES OF CIRCLE

WSNCE173211

(or) $= \frac{\pi}{4} d^2$ unit^2

Circumference of the circle $2\pi r$ (or) πd unit

Examples

1 Find the area of a circle whose radius is 1.54 m. Also find its circumference.

radius r = 1.54 cm

Area A = ?

Circumference C = ?

A $= \pi r^2$ unit^2

$= \frac{22}{7} \times 1.54 \times 1.54$

$= 7.4536$ m^2

C $= 2\pi r$ unit

$= 2 \times \frac{22}{7} \times 1.54$

= 9.68 m

2 Find out the circumference if the area of a circular shape of land is 616 m^2.

A $= \pi r^2$ unit^2

$r^2 = \frac{616}{\pi}$

$= \frac{616 \times 7}{22}$

$= 196$

$r = \sqrt{196}$

$= 14$ m

Circumference $= 2\pi r$ unit

$= 2 \times \frac{22}{7} \times 14$

= 88 m

3 Find the side of square into which it can be bent if a wire is in the form of a circle of radius 49 cm.

radius of circle r = 49 cm

side of square = ?

Perimeter of the square = Perimeter of the circle

$4a = 2\pi r$

$4a = 2 \times \frac{22}{7} \times 49$

$4a = 308$

$a = \frac{308}{4}$

= 77 cm

4 Find its radius if the difference between the circumference and diameter of a circle is 28 cm.

Circumference - Diameter = 28 cm

$2\pi r - d = 28$

$2\pi r - 2r = 28$

$2r(\pi - 1) = 28$

$2r(\frac{22}{7} - 1) = 28$

$2r(\frac{22-7}{7}) = 28$

$2r \times \frac{15}{7} = 28$

$r = \frac{28 \times 7}{15 \times 2}$

= 6.53 cm

5 What is the side of the largest square cut out from a circle of 50 cm dia.?

Diagonal of a square = Diameter of the circle

$\sqrt{2}a = 50$

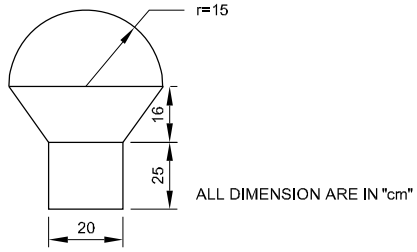
$a = \frac{50}{\sqrt{2}}$

$= \frac{50}{1.414}$

= 35.36 cm

6 Calculate the area of the figure given below.

Fig 2



PROPERTIES OF CIRCLE

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Area of rectangle = lb unit^2

= 25 x 20 cm^2

= 500 cm^2

Area of Trapezium = $\frac{1}{2}$ x (a + b) h

= $\frac{1}{2}$ x (30 + 20) 16 cm^2

= $\frac{1}{2}$ x 50 x 16 cm^2

= 400 cm^2

Area of Semi circle = $\frac{\pi r^2}{2}$ unit^2

= π x 15^2 x $\frac{1}{2}$ cm^2

= 353.57 cm^2

Total area of the figure = 500 + 400 + 353.57

= 1253.57 cm^2

7 Find the area of remaining steel plate if in a rectangular steel plate 16 cm x 12 cm, there are 6 holes each 4 cm in diameter.

Area of a rectangular plate = length x breadth unit^2

= 16 x 12

= 192 cm^2

No. of holes = 6

Radius of hole = 2 cm

Area of 6 holes = 6 x πr^2 unit^2

= 6 x $\frac{22}{7}$ x 2 x 2 unit^2

= 75.43 cm^2

Area of remaining plate = 192 - 75.43

= 116.57 cm^2

— — — — — — —

Semi circle

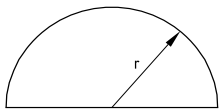
A semi circle is a sector whose central angle is 180º.
Length of arc of semi circle.

Length of arc ℓ = $2\pi r$ x $\frac{180}{360}$

= $2\pi r$ x $\frac{1}{2}$ = πr unit

Area of semi circle = $\frac{\pi r^2}{2}$ Sq. units

Fig 3



SEMI CIRCLE

WSNCE173213

Perimeter of a semi circle = $\frac{2\pi r}{2}$ + 2r

= πr + 2r

= r (π + 2) unit

Examples

1 Calculate the circumference and area of a semi circle whose radius is 6 cm.

radius r = 6 cm

Area A = ?

Circumference c = ?

A = $\frac{\pi r^2}{2}$ unit^2

= $\frac{22}{7}$ x $\frac{1}{2}$ x 6^2

Area (A) = $\frac{22}{7}$ x $\frac{1}{2}$ x 36

= $\frac{396}{7}$ = 56.57 cm^2

Perimeter of a semicircle = $\frac{2\pi r}{2}$ + 2r = πr + 2r

$= r(\pi + 2) = 6\left(\frac{22}{7} + 2\right)$

$= 6\left(\frac{22+14}{7}\right)$

$= 6 \times \frac{36}{7}$

$= \frac{216}{7}$

= 30.86 cm

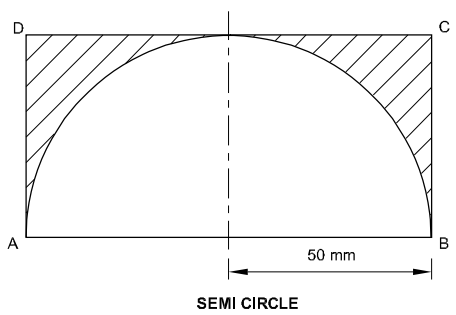
2 From the figure given below ABCD is a steel plate, a semi circular plate of radius 50 mm has been prepared by gas cutting. Find the waste area.

Plate length AB = 100 mm

Breadth BC = 50 mm

Radius = 50 mm

Fig 4



Waste area = Plate area - Area of semi circle

$= lb - \frac{\pi r^2}{2}$

$= 100 \times 50 - \frac{22 \times 50 \times 50}{7 \times 2}$

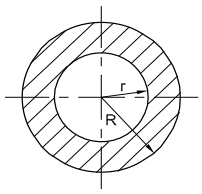
$= 5000 - 3928.57$

$= 1071.43\ mm^2$

— — — — — — —

Circular ring

Fig 5



R = Outer radius of circular ring

r = Inner radius of circular ring

Area of circular ring = $\pi (R^2 - r^2)\ unit^2$

or

$A = \pi (R + r)(R - r)\ unit^2$

1 Calculate the area of cross section of pipe having outside dia of 17 cm and inside dia of 14 cm.

Given:

Outer dia of pipe = 17 cm

Outer radius of pipe (R) = $\frac{17}{2}$ = 8.5 cm

Inner dia of pipe = 14 cm

Inner radius of pipe (r) = $\frac{14}{2}$ = 7 cm

To find:

Area of cross section of pipe = ?

Solution:

Area of cross section of pipe $= \pi (R + r)(R - r)\ unit^2$

$= \pi (8.5 + 7)(8.5 - 7)$

$= \frac{22}{7} \times 15.5 \times 1.5\ cm^2$

$= 73\ cm^2$

2 Find the distance between the boundaries and the area of the circular ring, if the circumference of two concentric circle are 134 cm and 90 cm.

Given:

Circumference of outer circle = 134 cm

Circumference of inner circle = 90 cm

To find:

Distance between the circles = ?

Area of circular ring = ?

Solution:

Circumference of outer circle = 134 cm

$2\pi R = 134$ cm

$R = \frac{134}{2\pi} = 21.32 cm$

Circumference of inner circle = 90 cm

$2\pi r = 90$ cm

$r = \frac{90}{2\pi} = 14.32$ cm

Distance between the circle = R - r

= 21.32 - 14.32 cm

= 7 cm

Area of circular ring $= \pi (R + r)(R - r)\ unit^2$

$= \pi (21.32 + 14.32)(21.32 - 14.32)\ cm^2$

$= \frac{22}{7} \times 35.64 \times 7\ cm^2$

$= 784.08\ cm^2$

3 A wire can be bend in the form of a circle of radius 56 cm. If it is bend in a form of a square, find the side.

Given:

Radius of circle = 56 cm

To find:

Side of square = ?

Solution:

Radius of circle = 56 cm

Circumference of circle = $2\pi r$ unit = 2π x 56 cm

Side of square = x cm

Wire can be bend from the form of round to square

Perimeter of square = circumference of circle

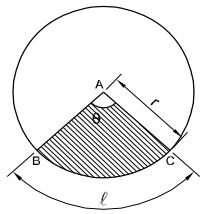
4 x a = 352 cm

$$a = \frac{352}{4} = 88 \text{ cm}$$

— — — — — — —

Sector of Circle

Fig 6



SECTOR OF CIRCLE

WSNCE173216

θ = Angle of sector of circle

l = Arc length

r = radius

Length of Arc $\ell = \frac{\theta}{360^0}$ x $2\pi r$ unit

Perimeter P = $2r + \ell$ unit

Area = $\frac{\theta}{360^0}$ x πr^2 unit^2 (or) A = $\frac{\ell r}{2}$ unit^2

1 Find the perimeter and area of a sector of circle of radius 7 cm and its angle is 120^0.

Given:

Angle of sector of circle = 120^0

Radius = 7 cm

To find:

Perimeter = ? , Area = ?

Solution:

Length of arc (ℓ) = $\frac{\theta}{360^0}$ x $2\pi r$ unit

= $\frac{120}{360}$ x 2 x $\frac{22}{7}$ x 7 cm

= 14.67 cm

Perimeter = $2r + \ell$ unit

= 2 x 7 + 14.67 cm

= 28.67 cm

Area = $\frac{\theta}{360^0}$ x πr^2 unit^2

Area = $\frac{120^\circ}{360^\circ}$ x $\frac{22}{7}$ x 7^2 cm^2 = 51.33 cm^2

2 Find the radius of the circle if the angle is 60^0 and the area of a sector of a circle is 144 cm^2,

Given:

Area of sector of circle (A) = 144 cm^2

Angle of sector of circle θ = 60^0

To find:

Radius of circle = ?

Solution:

Area (A) = $\frac{\theta}{360^0}$ x πr^2 unit^2

144 = $\frac{60^0}{360^0}$ x $\frac{22}{7}$ x r^2 cm^2

r^2 = 274.91 cm^2

r = $\sqrt{274.91}$ = 16.58 cm

3 Find the area of the sector whose angle is 105^0, and the perimeter of sector of circle is 18.6 cm.

Given:

Perimeter of a sector of a circle = 18.6 cm

Angle of sector of circle = 105^0

To find:

Area = ?

Solution:

Length of Arc (ℓ) = $\frac{\theta}{360^0}$ x $2\pi r$ unit

ℓ = $\frac{105^0}{360^0}$ x 2 x $\frac{22}{7}$ x r

= 1.83r

Perimeter (P) = ℓ + 2r unit

18.6 = 1.83r + 2r

3.83r = 18.6 cm

r = $\frac{18.6}{3.83}$ = 4.86 cm

Area A $= \frac{\theta}{360^0} \times \pi r^2$ unit^2

$= \frac{105^0}{360^0} \times \frac{22}{7}$ x (4.86) cm^2

$= 21.65$ cm^2

4 Find the area, if the radius is 12.4 cm and the perimeter of a sector of a circle is 64.8 cm.

Given:

Perimeter P = 64.8 cm

Radius r = 12.4 cm

To find:

Area A = ?

Solution:

Perimeter P $= \ell + 2r$ unit

$\ell = P - 2r$ unit

$= 64.8 - 2\ (12.4)$ cm

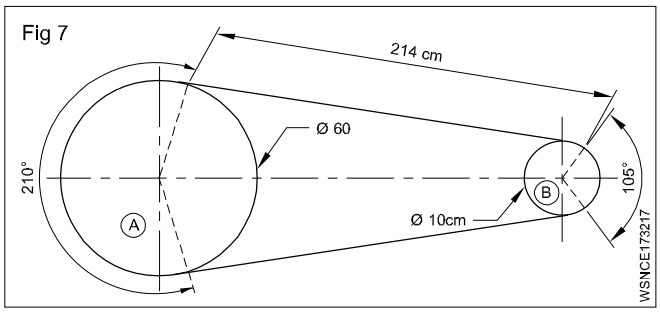
$= 64.8 - 24.8 = 40$ cm

Area A $= \frac{\ell r}{2}$ unit$^2 = \frac{40 \times 12.4}{2}$

$= 248$ cm^2

5 Find out the length of the belt , if the arrangement of a belt is shown in the figure below.

Fig 7



Solution:

Length $\ell_A = \frac{\theta}{360^0}$ x $2\pi r$ unit

$= \frac{210^0}{360^0}$ x 2 x $\frac{22}{7}$ x 30 = 110 cm

Length $\ell_B = \frac{\theta}{360^0}$ x $2\pi r$ unit

$= \frac{105^0}{360^0}$ x 2 x $\frac{22}{7}$ x 5 = 91.7 cm

$= \ell_A + \ell_B + 2$ x 214 cm

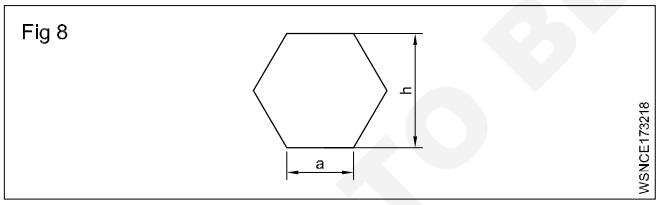
$= 110 + 9.17 + 428$ cm

$= 547.17$ cm

— — — — — — —

Hexagon

Fig 8



Side = a unit

Perimeter P = 6a unit

Area A = $6 \times \frac{\sqrt{3}}{4} \times a^2$ units^2 (Area of 6 equilateral triangle)

DAF (Distance Across Flats) = $\sqrt{3} \times a$ unit

DAC (Distance Across Corners) = 2 x a unit

1 Find out the perimeter, area, DAF and DAC of a regular hexagon whose side is 2cm.

(DAF - Distance Across Flats)

(DAC - Distance Across Corners)

Given: Side of hexagon (a) = 2cm

To Find: P = ?, A = ?, DAF = ?, DAC = ?

Solution:

Perimeter of hexagon (P) = 6a unit

= 6a unit = 6 x 2 cm = 12 cm

Area of hexagon A $= 6 \times \frac{\sqrt{3}}{4} \times a^2$ unit^2

$= 6 \times \frac{1.732}{4} \times 2^2$

$= 10.392$ cm^2

DAF (Distance Across Flats) $= \sqrt{3} \times a$ unit

$= \sqrt{3} \times 2 = 1.732$ x 2

$= 3.464$ cm

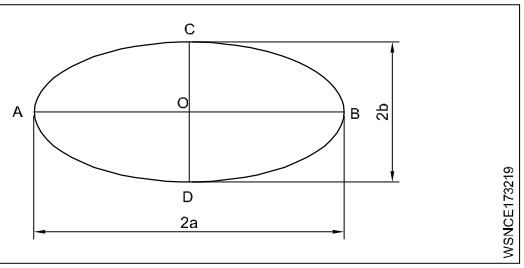
DAC (Distance Across Corners) = 2 x a unit

= 2 x 2 = 4 cm

— — — — — — —

Ellipse

Fig 9



Major axis AB = 2a

Half of Major axis OB = a,

Minor axis CD = 2b

Half of Minor axis OC = b

Area of ellipse A = π x a x b unit^2

Perimeter of ellipse P = $2\pi\sqrt{\frac{(a^2+b^2)}{2}}$ unit

1 Find its area and perimeter, if the major and minor axis of an ellipse are 12 cm and 8 cm respectively.

Solution:

Major axis 2a = 12 cm

a = $\frac{12}{2}$ = 6 cm

Minor axis 2b = 8 cm

b = $\frac{8}{2}$ = 4 cm

Area A = π x a x b unit^2

= $\frac{22}{7}$ x 6 x 4 cm^2

= 75.43 cm^2

Perimeter (P) = $2\pi\sqrt{\frac{(a^2+b^2)}{2}}$ unit

= $2\times\frac{22}{7}\sqrt{\frac{(6^2+4^2)}{2}}$ unit

= $2\times\frac{22}{7}\sqrt{\frac{36+16}{2}}$ unit

= $2\times\frac{22}{7}\times\sqrt{26}$

= 2 x $\frac{22}{7}$ x 5.1 = 32.06 cm

— — — — — — —

Assignment

Circle

1 Find the circumference and area of a circle whose radius is 10 metre.

2 Find its diameter if the area of a circle is 330 cm^2.

3 Find its area if the circumference of a circle is 50 cm.

4 Find out the area and circumference of a circle of diameter is 50 cm.

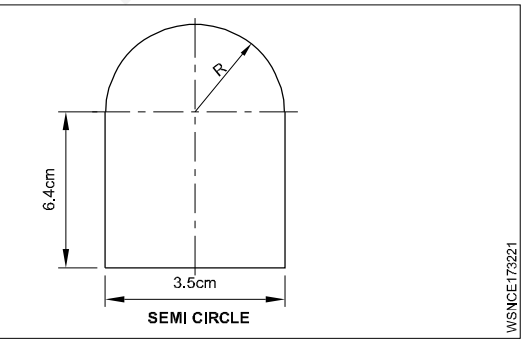
5 Find its area if the circumference of a circle is 44 cm.

Semi circle

1 Calculate the circumference and area of semi circle whose radius is 14 cm.

2 Find area of the given figure.

Fig 1



Circular ring

1 Find out area of a ring washer, whose inner radius and outer radius are 13 cm and 15 cm respectively.

2 Find the area of a ring portion of a washer whose outer dis is 30 m and inner dis is 20 m. Also calculate the difference between the circumference of circles.

Sector of circle

1 Find the perimeter and area of a sector of a circle of radius 5cm and its angle is 96^0.

2 Find the radius of the circle if the angle is 90^0 and the area of sector of a circle is 196 cm^2.

Hexagon

1 Find out the Area, perimeter, DAF, and DAC of hexagon of side 4cm.

2 Find the area of cross section of a regular hexagon rod whose side is 7.5 cm.

Ellipse

1 Find the area of the biggest ellipse that can be inscribed in a rectangle of length 18 cm and breadth 12 cm. Also calculate its perimeter.

2 How much fencing will be required to enclose an elliptical plot of ground the axes of the ellipse being 200 and 170 meter respectively.

Mensuration - Surface area and volume of solids - cube, cuboid, cylinder, sphere and hollow cylinder

Cube

All sides of cube are same i.e length,breadth and height have same value. It is bounded by six equal square faces.

Volume of cube = side x side x side

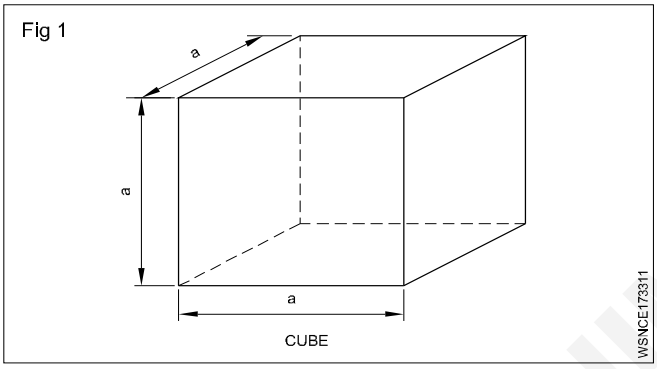
$= a^3$ unit^3

Lateral surface area $= 4a^2$ unit^2

Total surface area = 6 x side x side

$= 6a^2$ unit^2 $\qquad \sqrt{3} = 1.732$

Diagonal d $= \sqrt{3}a$



Rectangular solid (or) cuboid

Rectangular soild is bounded by six rectangular surfaces and opposite surfaces are equal and parallel to each other.

Volume of rectangular solid

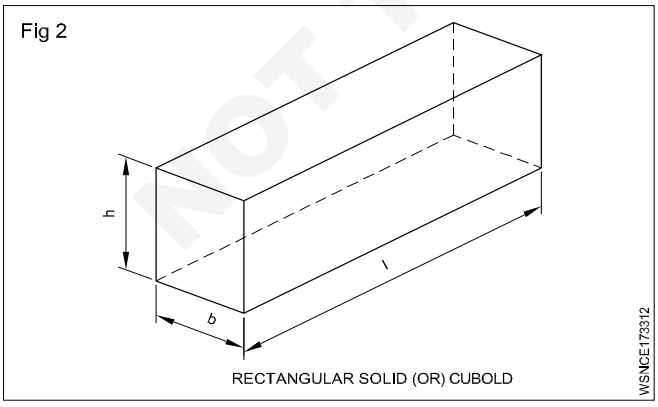
= Length x breadth x height

= l . b . h unit^3

Lateral surface area $= 2h(l+b)$ unit^2

Total surface area $= 2lb + 2bh + 2hl$

$= 2(lb+bh+hl)$ unit^2

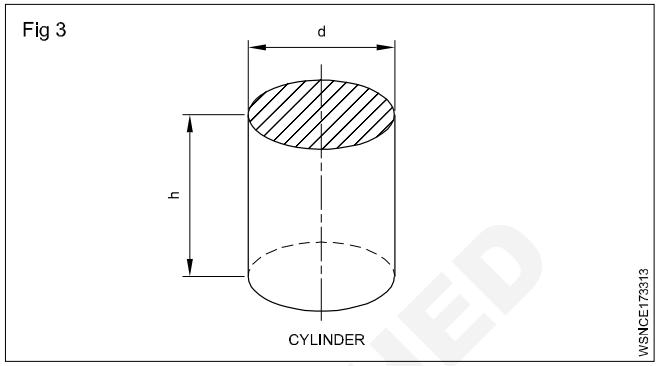


l = length, b = breadth and h = height

Cylinder

This is a prism whose top and bottom surfaces are equal and circular.

Volume of cylinder = $\pi r^2 h$ or $\frac{\pi}{4} d^2 h$



Curved area of cylinder = $2\pi rh$ unit^2

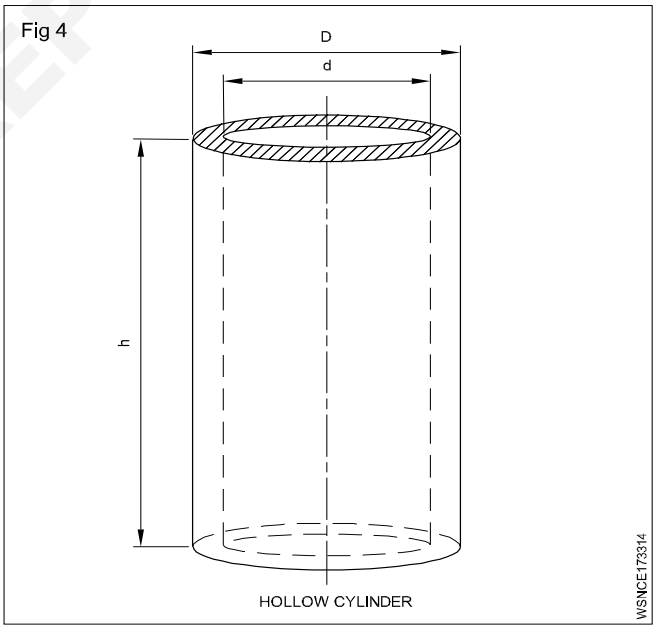
Total surface area of cylinder = $2\pi r(h+r)$ unit^2

r = Radius of base , d = Diameter of base

h = Height of cylinder

Hollow cylinder

Hollow means empty space. In hollow cylinder there is an empty place. Water pipe is an example of hollow cylinder.



Volume of hollow cylinder $= \pi (R^2 - r^2) h$ (or)

$= \pi (R + r) (R - r) h$ (or)

$= \frac{\pi}{4} (D^2 - d^2) h$ unit^3

$= \frac{\pi}{4}(D + d)(D - d)h$

Total surface area of hollow cylinder =

Inner + outer curved area + area of top and bottom circular part

$\therefore$ TSA : $2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

R = outer radius

r = inner radius

D = outer diameter

d = inner diameter

h = height of cylinder

t = thickness

Mean dia = $\frac{D-d}{2}$

If thickness given then:

Volume of hollow cylinder = π x mean dia x thickness x height

Finding out volumes of solids

The space occupied by a body is known its volume. The volume of a body indicates the capacity to hold substance in it.

The general form of Lateral surface area Total surface area and Volume is :

Lateral surface area = perimeter of the base x height

Total surface area = LSA + 2 (base area)

Volume = Area of base x height

Important and commonly used solids are described below one after another:

Cube

1 Find the diagonal, lateral surface area,, total surface area and volume of a cube of side 4.5 cm.

side a = 4.5 cm

diagonal d = $\sqrt{3}$a unit

= 1.732 x 4.5

= 7.794 cm

L.S.A = $4a^2$ unit^2

= 4 x 4.5 x 4.5

= 81 cm^2

T.S.A = $6a^2$ unit

= 6 x 4.5 x 4.5

= 121.5 cm^2

V = a^3 unit^3

= 4.5 x 4.5 x 4.5

= 91.125 cc.

2 Calculate volume of a cube where side is 9 cm

a = 9 cm

V = ?

V = a^3

= 9 x 9 x 9

= 729 cm^3

3 Find out side of the cube if a cube has volume of 3375cm^3.

V = 3375 cm^3

a = ?

a^3 = 3375

a = $\sqrt[3]{3375}$

= $\sqrt{3x3x3x5x5x5}$

= 3 x 5

= 15 cm

4 Find the side of a cube, if its surface area is 216 cm^2

Surface area = T.S.A = 216 cm^2

$6a^2$ = 216

a^2 = $\frac{216}{6}$

= 36

a = $\sqrt{36}$

= 6 cm

5 Find the side of the square tank, if its height is 2 metre and has the capacity to hold 50,000 litre of water.

Height of square shape tank (h) = 2 m

Capacity = 50,000 litre

1000 litre = 1 m^3

50,000 Litre = $\frac{50000}{1000}$

= 50 m^3

Capacity of tank = 50 m^3

a^2 x h = 50

a^2 x 2 = 50

a^2 = $\frac{50}{2}$ = 25 m^2

a = $\sqrt{25}$ = 5 m

Side of the square tank = 5 m

Rectangular Solid (or) Cuboid

1 Find its volume and T.S.A if a tank is 20 m long, 15 m broad and 12 m high.

l = 20m

b = 15 m

h = 12 m

v = ?

T.S.A = ?

Volume v = lbh unit^3

= 20 x 15 x 12

= 3600 m^3

T.S.A = 2(lb + bh + hl) unit^2

= 2((20 x 15) + (15 x 12) + (20 x 12))

= 2 (300 + 180 + 240)

= 1440 m^2

2 Find out its height if the cross section is 260 mm length and 180 mm wide rectangular and the capacity of a fuel tank is 10500 cm^3.

l = 260mm = 26 cm

b = 180 mm =18 cm

v = 10500 cm^3

h = ?

l.b.h = volume

26 x 18 x h = 10500

$$h = \frac{10500}{26 \times 18}$$

= 22.44 cm

3 How many litres of water it can store if a water tank has the following dimensions length = 1 metre, width = 0.8 metre and height = 1.2 metre?

Volume = l x b x h unit^3

= 1 x 0.8 x 1.2

= 0.96 m^3 [1 m^3 = 1000 litres]

= 0.96 x 1000

= 960 litres of water can store in the tank.

4 Find its volume if the base of a prism is a rectangle having 5m length, 4m breadth and the height of the prism is 15m.

The base of prism is rectangle

Area of base = length x breadth

= 5 x 4

= 20 square m

Volume of prism = Area of base x Height

= 20 x 15

= 300 cm^3

Cylinder

1 Find the volume and total surface are of a cylinder having 9cm diameter and 15 cm height.

diameter = 9 cm

radius r = 4.5 cm

height h = 15 cm

Volume V = ?

T.S.A = ?

V = $\pi r^2 h$ unit^3

$$= \frac{22}{7} \times 4.5 \times 4.5 \times 15$$

= 954.4 cm^3

T.S.A = $2\pi r(h+r)$ unit^2

$$= 2 \times \frac{22}{7} \times 4.5\ (15 + 4.5)$$

$$= 2 \times \frac{22}{7} \times 4.5 \times 19.5$$

= 551.4 cm^2

2 Calculate the radius if the curved surface area of a cylindrical roller is 48π cm^2 and the roller is 10 cm long

C.S.A = 48π cm^2

length = 10 cm

radius = ?

$2\pi rh = 48\pi$

$2 \times \pi \times r \times 10 = 48\pi$

$$r = \frac{48 \times \pi}{2 \times \pi \times 10}$$

= 2.4 cm

3 Find its radius if the volume of a cylinder is 5544 cm^3 and its height is 16 cm.

$\pi r^2 h = v$

$3.14 \times r^2 \times 16 = 5544$

$$r^2 = \frac{5544}{3.14 \times 16}$$

$$r^2 = \frac{5544}{50.24}$$

r^2 = 110.35

r = $\sqrt{110.35}$

= 10.5 cm

4 Find the diameter of the tank if the volume of a circular tank is 68.46 m^3, its height is 2 m.

$\pi r^2 h = 68.46$

$$r^2 = \frac{68.46}{3.14x2}$$

r^2 = 10.9

r = $\sqrt{10.9}$

= 3.3 m

diameter = 2r

= 2 x 3.3

= 6.6 m

5 A cylindrical vessel is to be made of 3 metre long and 1.9994 metre diameter. Calculate its total surface area, if it is in a closed form on one end.

$h = 3m$

$d = 1.9994\ m$

$r = 0.9997\ m$

T.S.A = C.S.A + Base area

$= 2\pi rh + \pi r^2$

$= (2 \times \frac{22}{7} \times 0.9997 \times 3) + (\frac{22}{7} \times 0.99997^2)$

$= 18.85 + 3.14$

$= 21.99\ m^2$

6 How many litres of water a cylinder of radius 75 cm and height 100 cm can hold.

$V = \pi r^2 h\ unit^3$

$= 3.142 \times 75 \times 75 \times 100$

$= 1767375\ cm^3$

$= \frac{1767375}{1000}$ [1000 cc = 1 litre]

= 1767.375 litres.

7 Calculate the height of cylindrical tin if a closed rectangular box 40 cm long, 30 cm wide and 25 cm deep has the same volume as that of cylinder tin of radius 17.5 cm.

Volume of cylinder = Volume of rectangular box

$\pi r^2 h = l \times b \times h$

$\frac{22}{7} \times 17.5 \times 17.5 \times h = 40 \times 30 \times 25$

$h = \frac{40 \times 30 \times 25 \times 7}{22 \times 17.5 \times 17.5}$

$= \frac{210000}{6737.5}$

= 31.17 cm

8 An oxygen cylinder is 15 cm in diameter and 100 cm in length. It is filled with gas under pressure so that every cm^3 of the cylinder contains 120 cm^3 of gas. How much cc of oxygen does this hold?

Volume of cylinder $= \pi r^2 h\ unit^3$

$= \frac{22}{7} \times 7.5 \times 7.5 \times 100$

$= 17678.57\ cm^3$

Gas contain in 1 cm^3 = 120 cm^3 of gas

Gas contain in 17678.57 cm^3 = 17678.57 x 120

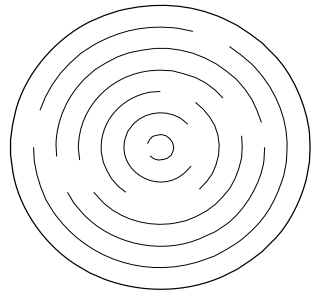
$= 2121428\ cm^3$

Volume of oxygen = 2121428 cc.

Sphere

Sphere is a solid circular body.

Fig 5



SPHERE

WSNCE173315

Volume of sphere $= \frac{4}{3}\pi r^3$ or

$= \frac{\pi}{6} d^3\ unit^3$

Total surface area of sphere $= 4\pi r^2\ unit^2$

Where r = Radius of sphere

d = Diametre of sphere

Radius = $\frac{1}{2}$ of diameter

1 Find the volume and surface area of a sphere of 3 cm radius.

$V = \frac{4}{3}\pi r^3\ unit^3$

$= \frac{4 \times 22 \times 3 \times 3 \times 3}{3 \times 7}$

$= 113.1\ cm^3$

Total Surface Area $= 4\pi r^2\ unit^2$

$= 4 \times \frac{22}{7} \times 3 \times 3$

$= 113.1\ cm^2$

2 Find the diameter of sphere having volume of 15625 cc.

$\frac{4}{3}\pi r^3 = Volume$

$\frac{4}{3} \times \frac{22}{7} \times r^3 = 15625$

$r^3 = \frac{15625 \times 3 \times 7}{4 \times 22}$

$$= \frac{328125}{88}$$

$$= 3728.69$$

$$r = \sqrt[3]{3728.69}$$

= 15.51 cm

diameter = 2 x radius

= 2 x 15.51

= 31.02 cm

3 How many spherical balls of 1 cm radius can be made from a sphere of 32 cm diameter.

No. of balls x volume of small sphere = Volume of bigger sphere

$$N \times \frac{4}{3} \times \pi r^3 = \frac{4}{3}\pi r^3$$

$$N \times \frac{4}{3} \times \pi \times 1^3 = \frac{4}{3} \times \pi \times r^3$$

$$N = 16 \times 16 \times 16$$

= 4096 balls

4 Three brass balls of diameters 3 cm, 4 cm and 5 cm are melted and make into one solid ball, if there is no wastage. Find the diameter of the solid ball.

1st ball d_1 = 3 cm,

r_1 = 1.5 cm

2nd ball d_2 = 4 cm,

r_2 = 2 cm

3rd ball d_3 = 5 cm,

r_1 = 2.5 cm

Diameter of new ball = ?

Volume of new ball = Volume of 3 spherical balls

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi (1.5^3 + 2^3 + 2.5)^3$$

$$r^3 = 3.375 + 8 + 15.625$$

$$r^3 = 27$$

$$r = \sqrt[3]{27}$$

$$r = \sqrt[3]{3x3x3}$$

$$r = 3 \text{ cm}$$

Diameter of the ball = 2 x r

= 2 x 3

= 6 cm

Assignment

Cube

1 Find the diagonal, lateral surface area, total surface area and volume of cube, whose side is 15 cm.

2 Find the volume of 10 cubes where each side is 5 cm.

3 Find its volume if a solid cube has each of its sides 60 mm long.

4 What is its side if the total surface area of a cube is 384 m^2.

Cuboid

1 Find the volume of the tank in m^3, if the length is 60 m, breadth 40 m and height 20 m.

2 Find the volume of a C.I. casting of a rectangular block having 25 cm x 20 cm x 8 cm size.

3 Calculate the total surface area of a box whose length, width and height are 120 cm, 50 cm and 60 cm respectively.

4 Find the volume of the sheet if a brass sheet is of 25 cm square and 0.4 cm thick.

Cylinder

1 Find the curved surface area of cylinder whose diameter is 18 cm and height 34 cm?

2 Find the total surface area of cylinder whose diameter is 24 cm and height 40 cm?

3 Find out the volume of cylinder whose base is 10 cm radius and height is 40 cm?

Sphere

1 Find the volume of sphere having diameter 3.5cm?

2 Find the total surface area of a sphere having radius 1.75 cm?

3 How many spherical balls of 1 cm radius can be made from a sphere of 16 cm diameter.

4 Three balls of diameter 2m, 4cm and 6 cm are melted and made into one solid ball. If there is no wastage, find the diameter of solid ball.

Mensuration - Finding the lateral surface area, total surface area and capacity in litres of hexagonal, conical and cylindrical shaped vessels

Hexagonal bar

Volume of Hexagonal bar = Area of hexagonal x height

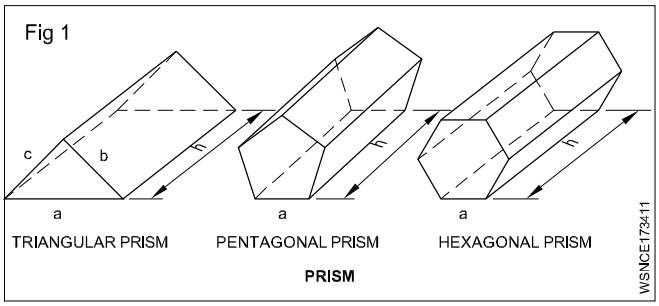
Lateral surface area of hexagonal bar

= 6 x side of hexagon x length of the bar

or = 3.464 x length of the bar x flat of hexagon

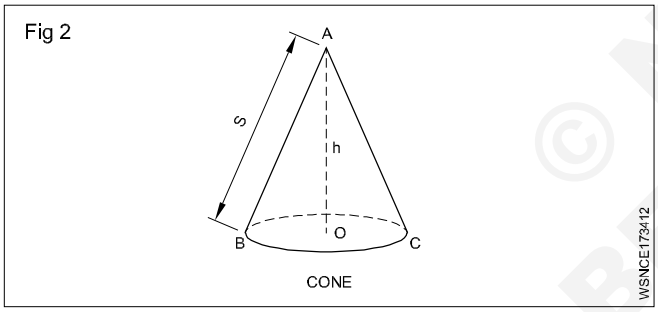
Total surface area of hexagonal bar

= lateral surface area + (2 x area of hexagon)



Cone

Cone is a pyramid with a circular base.



Volume of cone = $\frac{1}{3}\pi r^2 h$

or $= \frac{\pi}{12} d^2 h$

Curved area = πrs

Total surface area = $\pi r(s+r)$

Where r = radius of base

d = diametre of base

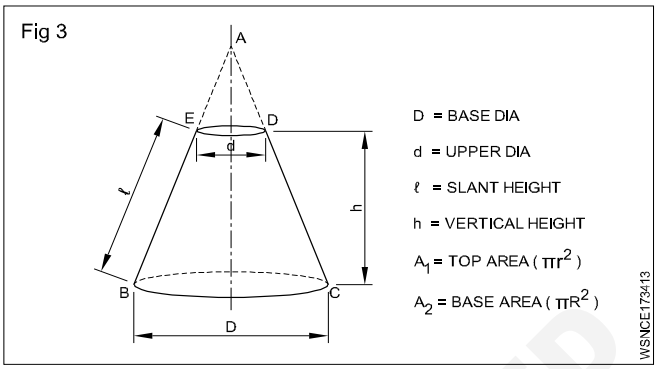
h = vertical height of cone

s = slant height $\sqrt{r^2 + h^2}$

Frustum of a cone

When a cone is cut by a plane parallel to the base, and upper part is removed, the formation appears, is termed as frustum of a cone. Buckets, oil cans etc.are such frustums in shape.

L.S.A = π l (R + r) unit^2



TSA = π l (R + r) + A_1 + A_2 unit^2

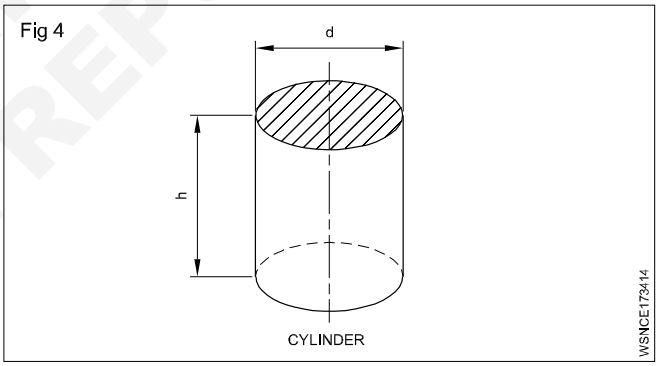
$V = \frac{\pi}{3} h (R^2 + Rr + r^2)$ unit^3

[A_1 = Top area ; A_2 = Bottom area]

Cylinder

This is a prism whose top and bottom surfaces are equal and circular.

Volume of cylinder = $\pi r^2 h$ or $\frac{\pi}{4} d^2 h$



Curved area of cylinder = $2\pi rh$

Total surface area of cylinder = $2\pi r(h+r)$

r = Radius of base, d = Diameter of base

h = Height of cylinder

Hollow cylinder

Hollow means empty space. In hollow cylinder there is an empty place. Water pipe is an example of hollow cylinder.

Volume of hollow cylinder = $\pi (R^2 - r^2) h$ (or)

$= \pi (R + r) (R - r) h$ (or)

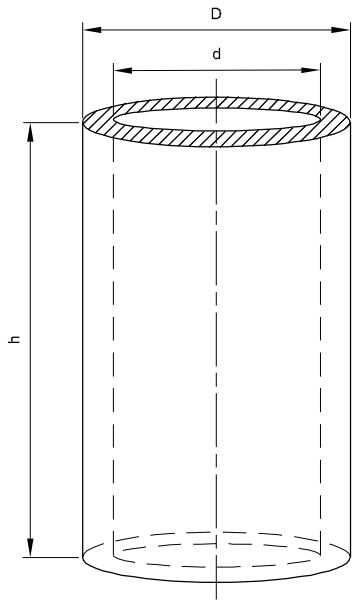
$= \frac{\pi}{4} (D^2 - d^2) h$

$= \frac{\pi}{4} (D + d)(D - d) h$

Total surface area of hollow cylinder =

Inner + outer curved area + area of top and bottom circular part

Fig 5



$\therefore$ TSA : $2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

R = outer radius

r = inner radius

D = outer diameter

d = inner diameter

h = height of cylinder

t = thickness

$$\text{Mean dia} = \frac{D - d}{2}$$

If thickness given then:

Volume of hollow cylinder = π x mean dia x thickness x height

Example

1 Find the volume of an hexagonal prism having its side 20 cm and height 200 cm.

Side of hexagonal prism (a) = 20 cm

Height (h) = 200 cm

Volume (V) = Base side area x Height

$= 6 \times \frac{\sqrt{3}}{4} \times a^2 \times h$

$= 6 \times \frac{\sqrt{3}}{4} \times 20 \times 20 \times 200$

$= 1,20,000 \times \sqrt{3}$

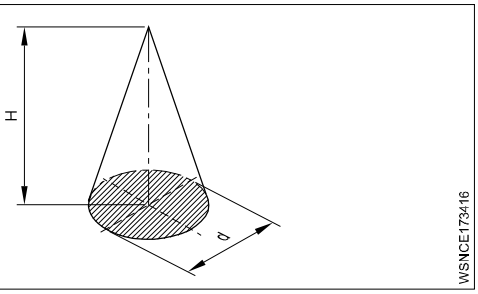
$= 1,20,000 \times 1.732$

$=$ **2,07,840 cm^3**

Volume of the hexagonal prism = 2,07,840 cm^3

2 Calculate the height. Also find the lateral surface area if a cone has a base diameter of 210 mm and its volume is 3056 cm3.

Fig 6



$$\text{Volume of a cone} = \frac{1}{3} \times \text{Area of base x height}$$

$$3056\ cm^3 = \frac{1}{3} \times 0.785 \times 210^2 mm^2 \times H$$

$$H = \frac{3056 \times 3 \times 1000 mm^3}{0.785 \times 210^2\ mm^2} = 264.82\ mm$$

$$L = \text{Slant height} = \sqrt{264.83^2 + 105^2} = 284.9mm$$

$$\text{Lateral surface area} = \frac{1}{2} \pi \times 210 \times 284.9 mm^2$$

= 94017 mm^2 = 940.17 cm^2

3 Determine its diameter in mm if the height of a rod of 1.6 metres and its volume is 1.017 metre^3.

$V = A \times H$

$V = \pi r^2 \times h$ (or) $\frac{\pi d^2}{4} \times h$

Volume = Area x Height

$= \frac{\pi d^2}{4} = 0.785\ d^2$

$1.017\ m^3 = 0.785 d^2 \times 1.6$ metres

$0.785 d^2 = \frac{1.017}{1.6} m^2$

$d^2 = \frac{1.017}{1.6 \times 0.785} m^2$

$= \frac{1.017}{1.6 \times 785} m^2$

$d = \sqrt{\frac{10170}{16 \times 785}}$ metre

$= \sqrt{\frac{10170}{12560}}$

$= \sqrt{0.8097}$

$= 0.8998$

$= 899.8mm$

Trigonometry - Measurement of angles

Introduction:

Trigonometry is the branch of mathematics which deals with the study of measurement and relationship of the three sides and three angles of a triangle.

Units:

Measurement of Angles

There are three systems of measuring the angle:

(i) Sexagesimal System

This is called British System. In this system, one right angle is divided into 90 equal parts which are called degrees.Each part is divided into 60 parts which are called minutes.Each minute is divided into 60 parts which are called seconds.The parts so divided respectively are called:

One degree (1°), one minute (1') and one second (1")

It means 1 right angle = 90° (90 degrees)

1 degree (1°) = 60' (60 minutes)

1 minute (1') = 60" (60 seconds)

In Trigonometry, mostly this system is used.

(ii) Centesimal System

This is called French System. In this system, the right angle is divided into 100 equal parts which are called grades. Each grade is divided into 100 minutes and each minute is divided into 100 seconds.

Parts so divided are respectively called:

One grade (1 g), one minute (1'), one second (1").

It means 1 right angle = 100 grades (100g)

1 grade (1 g) = 100 minutes (100')

1 minute (1') = 100 seconds (100")

90° = 100g (because each is a right angle)

This system is easier than Sexagesimal System. But to use this system many other systems will have to be devised that is why this system is not used.

(iii) Circular System

In this system, the unit of measuring angles is radian. It is that angle which is formed at the centre and is formed of an arc of length equal to radius in a circle.

There is one constant ratio between the circumference and dia of a circle. This is represented by π .

$$\frac{\text{Circumference}}{\text{Diameter}} = \text{constant point} = \pi$$

Circumference $= \pi \times$ dia

$= 2\pi r$ (where r is radius of the circle)

$$\pi = \frac{22}{7}$$

Circumference makes an angle $(2\pi r) = 360°$

Radius of the circle makes an angle (r) = 1 Radian

$$\text{ie : } \frac{C}{r} = \frac{360^\circ}{1\text{Radian}}$$

$$\frac{2\pi r}{r} = \frac{360^\circ}{1\text{Radian}}$$

$$2\pi = \frac{360^\circ}{1\text{Radian}}$$

2π Radian $= 360°$

π Radian $= 180°$

$$1 \text{ Radian} = \frac{180^\circ}{\pi}$$

$$1° = \frac{\pi}{180^\circ} \text{ Radian}$$

Examples

1 Convert 45°36'20" into degree and decimal of degree.

60 seconds = 1 minute

$$20 \text{ seconds} = \frac{20}{60} = 0.333'$$

60 minutes = 1 degree

$$36.333 \text{ minutes} = \frac{36.333}{60} = 0.606°$$

$45^0 36'20" = 45.606°$

2 Convert 24.59° into degree, minute and second

1 degree = 60 minutes

0.59 degree = 0.59 x 60 = 35.4'

1 minute = 60 seconds

0.4 minute = 60 sec x 0.4

= 24"

Therefore 24.59° = $24^0 35'24"$

3 Change $50^0 37'30"$ into degrees

By changing angle degrees into decimals

$$30" = \frac{30}{60} = 0.50'$$

37'30" = 37.5'

$$37.5' = \frac{37.5}{60} = 0.625^0$$

$50^0 37'30" = 50.625^0$

4 Convert $23^0 25' 32"$ into radians

We know $1^0 = 60' = 3600"$

Therefore $23^0 25' 32"$

$$= \left(23 + \frac{25}{60} + \frac{32}{3600}\right) degrees$$

$$= \frac{82800 + 1500 + 32}{3600}$$

$$= \frac{84332}{3600}$$

But $180^0 = \pi$ radians

Therefore 23.4255 degrees

$$= \frac{23.4255}{180} \pi \text{ radians}$$

$$= \frac{23.4255}{180} \times \frac{22}{7} \text{ radians}$$

= 0.4089 radians

5 Convert $87^0 19' 57"$ into Radian.

$$19'57" = 19' + \frac{57''}{60}$$

$$= 19' + 0.95'$$

$$= 19.95'$$

$$87°19.95' = 87° + \frac{19.95'}{60}$$

$$= 87° + 0.332° = 87.33°$$

$$1° = \frac{\pi}{180} \text{ radian}$$

$$87.33° = \frac{\pi}{180} \times 87.33 \text{ radian}$$

$$= 1.524 \text{ radian}$$

6 Convert $67°11'43"$ into Radian

$$11'43" = 11' + \frac{43''}{60}$$

$$= 11' + 0.716'$$

$$= 11.72'$$

$$67°11.72' = 67° + \frac{11.72'}{60}$$

$$= 67° + 0.195°$$

$$= 67.2°$$

$$1° = \frac{\pi}{180} \text{ radian}$$

$$67.2° = \frac{\pi}{180} \times 67.2 \text{ radian}$$

$$= 1.173 \text{ radian}$$

7 Convert $\frac{4}{7}\pi$ radian into degrees

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

$$\frac{4}{7}\pi \text{ radian} = \frac{180}{\pi} \times \frac{4}{7}\pi \text{ degree}$$

$$= 102.9 \text{ degree}$$

$$= 102^0 0.9 \times 60'$$

$$= 102^0 54'$$

8 Convert 0.8357 radian into degrees

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

$$0.8357 \text{ radian} = \frac{180}{\pi} \times 0.8357 \text{ degree}$$

$$= 47.88^0$$

$$= 47^0 0.88 \times 60'$$

$$= 47^0 52.80'$$

$$= 47^0 52'0.8 \times 60"$$

$$= 47^0 52'48"$$

9 Convert 2.752 radian into degrees

$$1 \text{ Radian} = \frac{180}{\pi} \text{ degree}$$

$$2.7520 \text{ radian} = \frac{180}{\pi} \times 2.752 \text{ degree}$$

$$= 157.7^0$$

$$= 157.7^0 \times 60'$$

$$= 157^0 42'$$

10 Convent $\frac{3}{5}\pi$ radian into degrees

$$1 \text{ Radian} = \frac{180}{\pi} \text{ degree}$$

$$\frac{3}{5}\pi \text{ radian} = \frac{180}{\pi} \times \frac{3}{5}\pi \text{ degree}$$

$$= 108^0$$

Assignment

Convert into Degree

1 12 Radian

Convert into Radians

2 78^o

3 $47^0 20'$

4 $52^0 36' 45"$

5 $25^0 38"$

Convert into degree, minute and seconds

6 46.723^o

7 68.625^o

8 0.1269 Radian

9 2.625 Radians

10 3/5 Radian

Trigonometry - Trigonometrical ratios

Dependency

The sides of a triangle bear constant ratios for a given definite value of the angle. That is, increase or decrease in the length of the sides will not affect the ratio between them unless the angle is changed. These ratios are trigonometrical ratios. For the given values of the angle a value of the ratios $\frac{BC}{AB}, \frac{AC}{AB}, \frac{BC}{AC}, \frac{AB}{BC}, \frac{AB}{AC}$ and $\frac{AC}{BC}$ do not change even when the sides AB, BC, AC are increased to AB', BC' and AC' or decreased to AB", BC" and AC".

For the angle

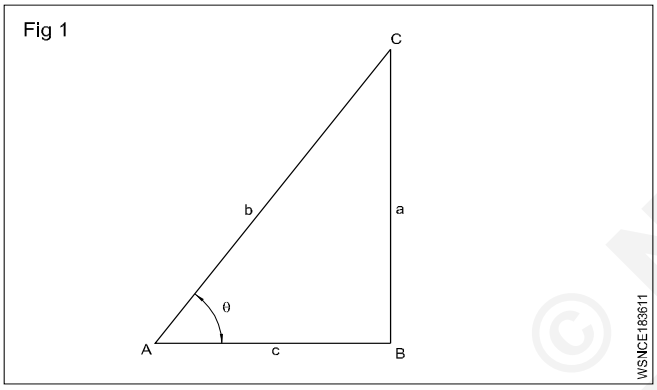
AC is the hypotenuse

AB is the adjacent side

BC is the opposite side.

The ratios

Fig 1



The six ratios between the sides have precise definitions.

$$\text{Sine } \theta = \frac{BC}{AC} = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \text{Sin } \theta$$

$$\text{Cosine } \theta = \frac{AB}{AC} = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \text{Cos } \theta$$

$$\text{Tangent } \theta = \frac{BC}{AB} = \frac{\text{Opposite side}}{\text{Adjacent side}} = \text{Tan } \theta$$

$$\text{Cosecant } \theta = \frac{AC}{BC} = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \text{Cosec } \theta$$

$$\text{Secant } \theta = \frac{AC}{AB} = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \text{Sec } \theta$$

$$\text{Cotangent } \theta = \frac{AB}{BC} = \frac{\text{Adjacent side}}{\text{Opposite side}} = \text{Cot } \theta$$

Relationship between the ratios

$$\text{Cosec } \theta = \frac{AC}{BC} = \frac{1}{\frac{BC}{AC}} = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{AC}{AB} = \frac{1}{\frac{AB}{AC}} = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{AB}{BC} = \frac{1}{\frac{BC}{AB}} = \frac{1}{\tan \theta}$$

$$\sin \theta = \frac{\text{side } BC}{\text{side } AC} = \frac{a}{b}$$

$$\cos \theta = \frac{\text{side } AB}{\text{side } AC} = \frac{c}{b}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{\frac{a}{b}}{\frac{c}{b}} = \frac{a}{b} \times \frac{b}{c} = \frac{a}{c}$$

$$= \frac{\text{side } BC}{\text{side } AB} = \tan \theta$$

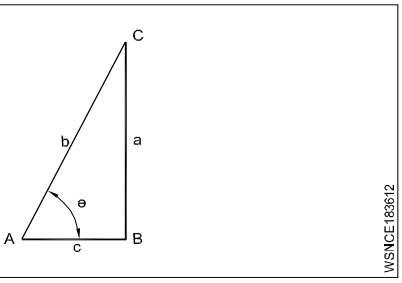
$$\sin \theta = \frac{1}{\text{cosec } \theta} \text{ or cosec } \theta = \frac{1}{\sin \theta} \text{ or } \sin \theta . \text{cosec } \theta = 1$$

$$\cos \theta = \frac{1}{\sec \theta} \text{ or } \sec \theta = \frac{1}{\cos \theta} \text{ or } \cos \theta . \sec \theta = 1$$

$$\tan \theta = \frac{1}{\cot \theta} \text{ or } \cot \theta = \frac{1}{\tan \theta} \text{ or } \cot \theta . \tan \theta = 1$$

By pythogoras theorem we have, $AC^2 = AB^2 + BC^2$

Fig 2



Dividing both sides of the equation by AC^2, we have

$$\frac{AC^2}{AC^2} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$= \left[\frac{AB}{AC}\right]^2 + \left[\frac{BC}{AC}\right]^2$$

$1 = (\cos \theta)^2 + (\sin \theta)^2$

$\sin^2 \theta + \cos^2 \theta = 1$

Sine, Cosine, Tangent, Cosec, Sec and Cotangent are the six trigonometrical ratios

$\tan \theta = \frac{\text{Sin } \theta}{\text{Cos } \theta}$ and $\sin^2 \theta + \cos^2 \theta = 1$

$\sin^2\theta + \cos^2\theta = 1$

It can be transformed as

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$\text{or } \cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$$

We know $\sin^2\theta + \cos^2\theta = 1$

Dividing both sides by $\cos^2\theta$.

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\text{or } 1 + \tan^2\theta = \sec^2\theta$$

Using the same equation

$$\sin^2\theta + \cos^2\theta = 1.$$

Dividing both sides by $\sin^2 q$,

$$\frac{\text{Sin}^2\theta}{\text{Sin}^2\theta} + \frac{\text{Cos}^2\theta}{\text{Sin}^2\theta} = \frac{1}{\text{Sin}^2\theta}$$

$$1 + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2 q = \text{cosec}^2 q$$

$$1 + \tan^2 q = \sec^2 q$$

Trigonometrical Tables

Ratio	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

When q increases,

Sine value increases;

Cosine value decreases;

Tangent value increases to more than 1 when the angle is more than 45° (tan60° = 1.732)

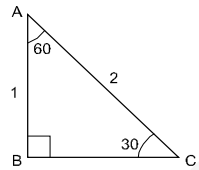
Sine of an angle =	**Cosine of its complementary angle**
Cosine of an angle =	**Sine of its complementary angle**

Examples

If $\sin 30^0 = \frac{1}{2}$ find the value of $\sin 60^0$

By applying pythagores theorem

Fig 3



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$BC^2 = AC^2 - AB^2$

$BC^2 = 2^2 - 1^2$

$= 4 - 1$

$= 3$

$BC = \sqrt{3}$

$\sin 60^0 = \frac{\sqrt{3}}{2}$

$\text{Cos}\theta = \frac{3}{5}$ Find the other trigonometrical ratios

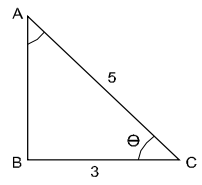
By applying pythagores theorem

$AB^2 = AC^2 - BC^2$

$= 5^2 - 3^2 \quad = 25 - 9$

$= 16$

Fig 4



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$AB = \sqrt{16} = 4$

Now $\sin\theta = \frac{4}{5}$

$\tan\theta = \frac{4}{3}$

$\text{Cosec } \theta = \frac{5}{4}$

$\sec\theta = \frac{5}{3}$

$\cot\theta = \frac{3}{4}$

Signs of trigonometrical functions for angles more than 90^0

Ratio	90 - θ	90 + θ	180 - θ	180 + θ	270 - θ	270 + θ	360 - θ	- θ
sin	cos	cos	sin	- sin	- cos	- cos	- sin	- sin
cos	sin	- sin	- cos	- cos	- sin	sin	cos	cos
tan	cot	- cot	- tan	tan	cot	- cot	- tan	- tan
cosec	sec	sec	cosec	- cosec	- sec	- sec	- cosec	- cosec
sec	cosec	- cosec	- sec	- sec	- cosec	cosec	sec	sec
cot	tan	- tan	- cot	cot	tan	- tan	- cot	- cot

Simplify :

$\cot\theta + \tan(180+\theta) + \tan(90-\theta) + (\tan 360 - \theta)$

$= \cot\theta + \tan\theta - \cot\theta - \tan\theta$

$= 0$

Simplify :

$$\frac{\cos(90+\theta)\sec(-\theta)\tan(180-\theta)}{\sec(360-\theta)\sin(180+\theta)\cos(90-\theta)}$$

$$= \frac{(-\sin\theta)\times(\sec\theta)\times(-\tan\theta)}{(\sec\theta)\times(-\sin\theta)\times(-\sin\theta)}$$

$$= \frac{\tan\theta}{\sin\theta} = \frac{1}{\cos\theta} = \sec\theta$$

simplify:

$$\frac{\cos(90^\circ+\theta)\sec(-\theta)\tan(180^\circ-\theta)}{\sec(360^\circ-\theta)\sin(180^\circ+\theta)\cot(90^\circ-\theta)}$$

$\cos(90^0 + \theta) = -\sin\theta$

$\sec(-\theta) = \sec\theta$

$\tan(180^0 - \theta) = -\tan\theta$

$\sec(360^0 - \theta) = \sec\theta$

$\sin(180^0 + \theta) = -\sin\theta$

$\cot(90^0 + \theta) = -\tan\theta$

$$\frac{\cos(90^\circ+\theta)\sec(-\theta)\tan(180^\circ-\theta)}{\sec(360^\circ-\theta)\sin(180^\circ+\theta)\cot(90^\circ-\theta)}$$

$$= \frac{(-\sin\theta)(\sec\theta)(\tan\theta)}{(\sec\theta)(-\sin\theta)(-\tan\theta)}$$

$= 1$

Simplify:

$\cot\theta + \tan(180^0 + \theta) + \tan(90^0 + \theta) + \tan(360^0 - \theta)$

$\tan(180^0 - \theta) = \tan\theta$

$\tan(90^0 + \theta) = -\cot\theta$

$\tan(360^0 - \theta) = -\tan\theta$

$\cot\theta + \tan(180^0 + \theta) + \tan(90^0 + \theta) + \tan(360^0 - \theta)$

$\cot\theta + \tan\theta - \cot\theta - \tan\theta = 0$

Assignment

1 Given $\sin 30^0 = 1/2$, find the value of $\tan 60^0$

2 If $\cos\theta = 4/5$, find the other radios

3 If sin A = 3/5, find $\cos\theta$, $\tan\theta$ & $\sec\theta$

4 If $\tan\theta = 24/7$, find $\sin\theta$ and $\cos\theta$

5 Find the value of $\cos\theta$ and $\tan\theta$, if $\sin\theta = 1/2$

6 If $\cos\theta = 5/13$, find the value of $\tan\theta$

7 If $\sin\theta = 1/2$, find the value of $\sin^2\theta - \cos^2\theta$

8 What is the value of

$$\frac{\sin^2 30^\circ}{\cos^2 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$$

Simplify :

1 $\tan(90 + A) + (\tan 180 + A)\tan(90 + A)$

2 $$\frac{\cos(90+\theta)\cdot\sec(-\theta)\cdot\tan(180-\theta)}{\sec(360+\theta)\cdot\sin(180+\theta)\cdot\cot(90+\theta)}$$

Trigonometry - Trigonometrical tables

Use of trigonometrical tables (Ref: Sin, Cos & Tan Table)

	Minutes from 0 to 4						Mean difference				
Deg.	0'	6'	12'	18'	24'	54'	1'	2'	3'	4'	5'
0				.				.			
1				.				.			
2				.				.			
3				.				.			
.				.				.			
.				.				.			
.				.				.			
26	..	..	..	x	..	..	..	5			
.											
.											
.											
89											

Sine value for 26°– 20'

Refer to Natural sine table.

Degrees column go up to 26° down

Minutes column 18' horizontal and under this note the value which is given as 0.4431.

Under mean difference for 2' in the same horizontal line 5 is given. Add this to the extreme right number noted for 26°– 18'.

Sine 26° – 20' = 0.4431 + .0005 = 0.4436

Cosine value for 43° – 41'

Referring to the Natural cosines table for 43° –36' it is given as 0.7242 and the mean difference for 5' minutes is given as 10.

cos 43°– 41' = value for cos 43°.36 '

– the value given for

mean difference of 5 ' = 0.7242 – 0.0010

= 0.7232

When reading sine value add the mean difference value. When reading cosine value subtract the mean difference value.

Arrangement

Values of trigonometrical ratios can be taken from mathematical tables.

The left hand vertical column consists of degrees.

The top horizontal column is arranged in minutes in steps of 6' from 0' to 54'. In the extreme right horizontal columns the mean differences are written in minutes from 1' to 5' in steps of 1' to account for angles with minutes between the interval of 6'.

- **The values of cosine, cosecant and cotangent decrease when the value of the angle increases.**
- **For sine, secant and tangent, the value increases when the angle increases.**
- **The value of sine and cosine will never be more than 1.**
- **The value of secant and cosecant will never be less than 1.**
- **The value of Tan and Cot ranges from 0 to ∞.**

EXAMPLE

From the tables obtain the cosine of 45°20 '.

cos 45°18 ' = 0.7108

mean difference for 2 ' = 0.0004

cos 45° 20 ' = 0.7104

— — — — — — —

SINE TABLE

1 Sin 25^0 = 0.4226

2 Sin 17^0 5'

sin 17°	=	0.2924
Difference 5'	=	14
sin 17° 5'	=	0.2938 Ans

3 **sin 17° 45' 13"**

sin 17° 46'	=	0.3051
sin17° 45'	=	0.3048
Difference 1'	=	0.0003

1' (or) 60" = 0.0003

13" = $\frac{0.0003}{60} \times 13$

= $\frac{0.0039}{60}$

= $\frac{0.00039}{6}$

= 0.000065

$\sin 17^0 45'$ = 0.3048

13" = 0.000065

$\sin 17^0 45'13"$ = 0.304865 Ans.

4 $\sin 82^\circ$ 14' 18"

$\sin 82^\circ$ 15' = 0.9908

$\sin 82^\circ$ 14' = 0.9908

Difference 1' = 0

1' (or) 60" = 0

sin18" = 0

$\sin 82^\circ$ 14' = 0.9908

18" = 0.0000

$\sin 82^\circ 41'18"$ = 0.9908 Ans.

Finding the corresponding angles when sine values are given:

1. Sin θ = 0.9925

θ = 83^0

2. Sin θ = 0.8791

0.8788 = Sin 61^0 30'

0.0003 = 2'

0.8791 = Sin 61^0 32'

3. sin θ = 0.68015

0.6794 = sin 42^0 48'

0.0006 = 3'

0.6800 = sin 42^0 51'

0.6803 = sin 42^0 52'

Difference 0.0003 = 1' (or) 60"

0.00015 = $\frac{60}{0.0003}$ X 0.00015

= $\frac{60 \times 15}{30}$

= 30"

0.6800 = sin 42^0 51'

0.00015 = 30"

0.68015 = sin 42^0 51'30"

θ = 42^0 51'30"

Calculations involving tapers

D - Big diameter of the taper

d - small diameter of the taper

C - Taper Ratio - 1:x

Fig 1

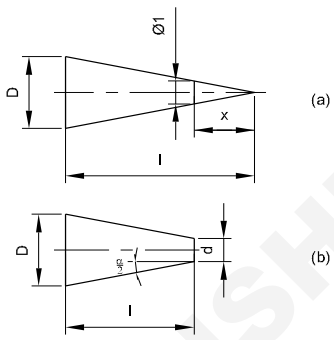

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Fig 2

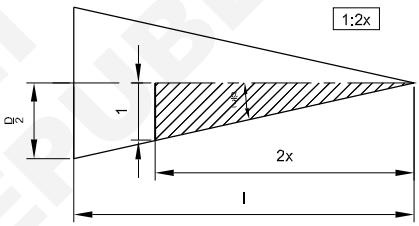

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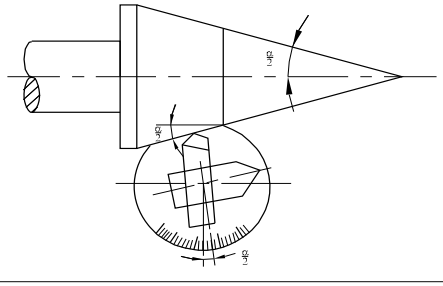
Fig 3



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$\frac{C}{2}$ Ratio of inclination - 1:2 **x**

l - length of taper

a - included angle of taper

$\frac{\alpha}{2}$ - setting angle

Taper ratio = Ratio of inclination (for wedges).

Taper ratio

The ratio between the difference in diameter to the length of the taper is known as taper ratio. *D* is the difference in larger diameter shown in the sketch as the small diameter of taper is 0. Taper ratio is *D : l* . In the sectioned portion the difference in diameter is 1 and the length of taper is shown as *x* .

C = D: d = 1 : x as per Fig 1 (a), $C = \frac{D-d}{l}$ as per Fig 1 (b)

Ratio of inclination

Taking half of the taper, $\frac{D}{2}$ is the difference in diameter for a taper length of l, if d = 0.

$\therefore \frac{C}{2} = \frac{D}{2l}$ if the small diameter is 0

or $\frac{C}{2} = \frac{D-d}{2l}$

1 Ratio of inclination = $\frac{1}{2}$ of the taper ratio.

Setting angle

One of the methods of turning taper is by swivelling the compound slide to an angle known as setting angle and feeding the tool at an angle to the axis of work.

$$\tan\frac{\alpha}{2} = \frac{C}{2} = \frac{D-d}{2l}$$

$$\tan\frac{\alpha}{2} = \frac{\text{taper ratio}}{2}$$

$$= \frac{\text{difference in diameter}}{2 \times \text{taper length}}$$

NOTE: $\frac{\alpha}{2}$ is the setting angle which is equal to half of the included angle of the taper.

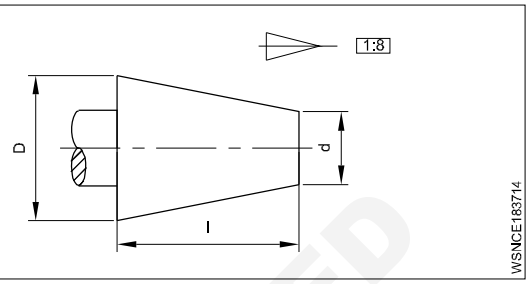
Taper Ratio C = 1 : x or D : l or $(D-d)$: l

Ratio of inclination $\frac{C}{2} = 1 : 2x = \frac{D}{2} : l$ or $\left(\frac{D-d}{2}\right) : l$

Setting angle determination is by the formula

$$\tan\frac{\alpha}{2} = \frac{D-d}{2l} = \frac{C}{2}$$

Fig 4



EXAMPLE

A pivot in the form of a frustum of a cone has a taper ratio 1:8. If the small diameter is 30 mm and length of taper is 80 mm, find its large diameter.

$C = 1:8 = \frac{1}{8}$

$\therefore \frac{D-d}{l} = \frac{1}{8}$

$\therefore D - d = \frac{1}{8} = \frac{80}{8} = 10\,\text{mm}.$

D – 30 mm = 10 mm

D = 10 mm + 30 mm = 40 mm

Large diameter D = 40 mm

— — — — — — — —

Cos Table

1 Cos 38°

Cos 38° = 0.7880

2 Cos 83°12'

Cos 83°12' = 0.1184

3 Cos 26°40'

Cos 26°36' = 0.8942

4' = 5 (-)

Cos 26°40' = 0.8937

---

4 Cos 31°20'

Cos 31°18' = 0.8545

2' = 3 (-)

Cos 31°20' = 0.8542

Find the corresponding angles when cos values are given:

1 Cos θ = 0.5150

θ = 59°

2 Cos θ = 0.0192

θ = 88°54'

3 Cos θ = 0.9682

0.9686 = cos 14°24'

(-) 4 = 5'

0.9682 = cos 14°29'

θ = 14°29'

4 Cos θ = 0.8476

0.8480 = cos 32° 0'

(-) 0.0003 = 2'

0.8477 = cos 32° 2'

0.8475 = cos 32° 3'

0.0002 = 1' (or) 60"

$$0.0001 = \frac{}{60} \times 0.0001$$

$$= \frac{0.0002}{60}$$

$$= \frac{}{2} \times 1$$

= 30"

0.8477 = Cos 32° 2'

(-) 0.0001 = 30" (+)

0.8476 = Cos 32° 2' 30"

---

tan Table

1 tan 35° 37'

tan 35° 36' = 0.7159

1' = 0.0004

---

tan 35° 37' = 0.7163

---

2 tan 50° 5'

Tan 50° 0' = 1.1918

5' = 0.0036

Tan 50° 5' = 1.1954

Find the corresponding angles when tan values are given

1 tan θ = 0.3972

0.3959 = tan 21° 36'

0.0013 = 4'

0.3972 = tan 21° 40'

---

2 tan θ = 1.0065

1.0035 = tan 45° 6'

0.0030 = 5'

1.0065 = tan 45° 11'

Problems Related with Trigonometrical tables

1 A 250 mm Sine bar is used to measure an angle. If the difference in height is 5 cm, find the angle.

$$Sin\theta = \frac{\text{Opp.side}}{\text{Hyp.}} = \frac{h}{l}$$

$$= \frac{5 \text{ cm}}{250 \text{ mm}}$$

$$= \frac{50 \text{ mm}}{250 \text{ mm}}$$

= 0.2000

θ = 11° 32'

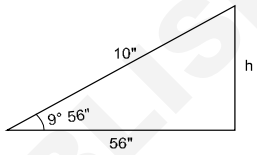
2 Find the height of the slip gauge if a Sine bar with plugs of 10" centre is set up to inspect a taper having an included angle of 9° 56".

$$Sin\theta = \frac{\text{Opp.side}}{\text{Hyp.}} = \frac{h}{l}$$

$$\text{Sin } 9°56" = \frac{h}{10}$$

$\therefore h = 10 \times \sin 9°56"$

Fig 5



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Sin 9° 0" = 0.1564

Sin 9° 1" = 0.1567

1' (or) 60" = 0.0003

$$56" = \frac{0.0003}{60} \times 56$$

$$= \frac{0.0168}{60}$$

$$= \frac{0.00168}{6}$$

= 0.00028

sin9° 0' 00" = 0.1564

56" = 0.00028

---

sin9° 0' 56" = 0.15668

---

h = 10 x sin9° 56"

= 10 x 0.15668

= 1.5668 cm

Height of slip gauge = 1.5668"

3 Find the angle which the ladder makes with the ground if the foot of a 4.5 m long ladder is placed at 1 m away from the wall.

In Right angled D

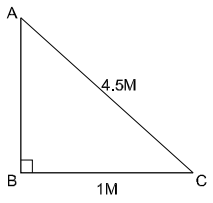
$$\text{Cos C} = \frac{BC}{AC}$$

$$\text{Cos}\ \theta = \frac{1\ m}{4.5\ m} = 0.2222$$

	Cos	=	0.2222
	0.2233	=	cos 77° 6'
(-)	0.0011	=	4' (+)
	0.2222	=	cos 77° 10'

θ = 77°10'

Fig 6



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Assignment

I Find the values of the given angles

1 Sin 65°
2 Sin 42°23'
3 Sin 66°35' 32"
4 Sin 7°15' 41"
5 Sin 27°27"
6 Cos 26°40'
7 Cos 47°39'
8 Cos 79°31' 53"
9 Tan 28°45'
10 Tan 67°27' 36"

II Find corresponding angles for given values

1 Sin θ = 0.3062
2 Sin θ = 0.6002
3 Sin θ = 0.22453
4 Sin θ = 0.04802
5 Cos θ = 0.6446
6 Cos θ = 0.8926
7 Cos θ = 0.11773
8 Cos θ = 0.21646
9 Tan θ = 0.3411
10 Tan θ = 2.3868

III

1 Calculate its base. if the slant height of a cone is 12.25 cm and the vertex angle is 110^0.
2 A ladder 2.5 m long makes an angle of 60^0 with the ground. Find the height of the wall where the ladder touches the wall.
3 A sine bar of 200 mm is to be set at an angle of $15^0 15'3"$. Select the slip gauge block to built up the required height.
4 In a right angled triangle ABC, ∠C = 90°, If AB = 50 mm and ∠B = 75°, Find the remaining sides.
5 Calculate the required length of the bar for this point if a centre point having an included angle of 60° is to be turned at the end of a 50 mm dia bar.

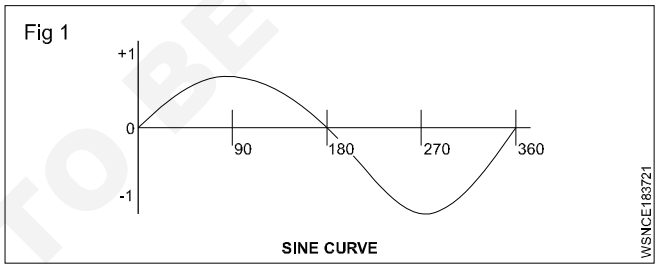
Natural Sines

°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	3	6	9	12	15
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	3	6	9	12	15
2	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506	3	6	9	12	15
3	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680	3	6	9	12	15
4	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854	3	6	9	12	14
5	0.0872	0.0899	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028	3	6	9	12	14
6	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201	3	6	9	12	14
7	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374	3	6	9	12	14
8	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547	3	6	9	11	14
9	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719	3	6	9	11	14
10	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891	3	6	9	11	14
11	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062	3	6	9	12	14
12	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232	3	6	9	11	14
13	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402	3	6	8	11	14
14	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571	3	6	8	11	14
15	0.2558	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740	3	6	8	11	14
16	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907	3	6	8	11	14
17	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074	3	6	8	11	14
18	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239	3	6	8	11	14
19	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404	3	5	8	11	14
20	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567	3	5	8	11	14
21	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730	3	5	8	11	14
22	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891	3	5	8	11	13
23	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051	3	5	8	11	13
24	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210	3	5	8	11	13
25	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368	3	5	8	11	13
26	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524	3	5	8	10	13
27	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679	3	5	8	10	13
28	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833	3	5	8	10	13
29	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985	3	5	8	10	13
30	0.500	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135	3	5	8	10	13
31	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284	2	5	7	10	12
32	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432	2	5	7	10	12
33	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577	2	5	7	10	12
34	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721	2	5	7	10	12
35	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864	2	5	7	9	12
36	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004	2	5	7	9	12
37	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143	2	5	7	9	12
38	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280	2	5	7	9	11
39	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414	2	4	7	9	11
40	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547	2	4	7	9	11
41	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678	2	4	7	9	11
42	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807	2	4	6	9	11
43	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0,6921	0.6934	2	4	6	8	11
44	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059	2	4	6	8	10
45	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181	2	4	6	8	10
46	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302	2	4	6	8	10
47	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420	2	4	6	8	10
48	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536	2	4	6	8	10
49	0.7547	0.7558	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649	2	4	6	8	9
50	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760	2	4	6	7	9
51	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869	2	4	5	7	9
52	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976	2	4	5	7	9
53	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080	2	3	5	7	9
54	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181	2	3	5	7	8
55	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281	2	3	5	7	8
56	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377	2	3	5	6	8
57	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471	2	3	5	6	8
58	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563	2	3	5	6	8
59	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652	1	3	4	6	7

Natural Sines

°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
60	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738	1	3	4	6	7
61	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821	1	3	4	6	7
62	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902	1	3	4	5	7
63	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980	1	3	4	5	6
64	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056	1	3	4	5	6
65	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128	1	2	4	5	6
66	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198	1	2	3	5	6
67	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265	1	2	3	4	6
68	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330	1	2	3	4	5
69	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391	1	2	3	4	5
70	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449	1	2	3	4	5
71	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505	1	2	3	4	5
72	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558	1	2	3	3	4
73	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608	1	2	2	3	4
74	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655	1	2	2	3	4
75	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699	1	1	2	3	4
76	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740	1	1	2	3	3
77	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778	1	1	2	2	3
78	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813	1	1	2	2	3
79	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845	1	1	2	2	3
80	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874	0	1	1	2	2
81	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900	0	1	1	2	2
82	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923	0	1	1	1	2
83	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943	0	1	1	1	2
84	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960	0	1	1	1	1
85	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974	0	0	1	1	1
86	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985	0	0	1	1	1
87	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993	0	0	0	1	1
88	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998	0	0	0	0	0
89	0.9998	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0
90	1.0000														

Fig 1



SINE CURVE

Quadrant	Angle	sinA =	Examples
First	0 to 90°	sin A	sin 34°38' = 0.5683
Second	90° to 180°	sin(180° – A)	sin 145°22' = sin(180° – 145° 22') = sin 34°38' = 0.5683
Third	180° to 270°	–sin(A – 180°)	sin 214°38' = –sin(214°38' – 180°) = –sin34°38' = –0.5683
Fourth	270° to 360°	–sin(360° – A)	sin 325°22' = –sin(360° – 325°22') = – sin 34°38' = –0.5683

Natural Cosines

Numbers in different columns to be subtracted**, not added**

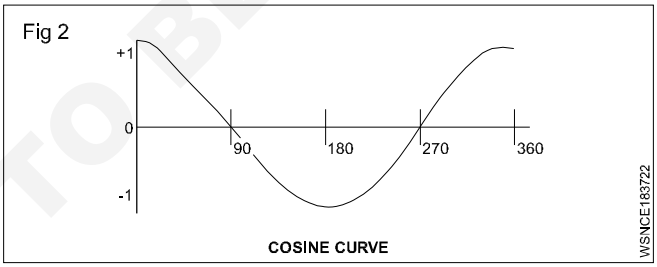
°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0	0	0	0	0
1	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995	0	0	0	0	0
2	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987	0	0	0	1	1
3	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977	0	0	1	1	1
4	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963	0	0	1	1	1
5	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947	0	1	1	1	1
6	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928	0	1	1	1	2
7	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905	0	1	1	1	2
8	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880	0	1	1	2	2
9	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851	0	1	1	2	2
10	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820	1	1	2	2	3
11	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785	1	1	2	2	3
12	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748	1	1	2	2	3
13	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707	1	1	2	3	3
14	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664	1	1	2	3	4
15	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617	1	2	2	3	4
16	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568	1	2	2	3	4
17	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516	1	2	3	3	4
18	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461	1	2	3	4	5
19	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403	1	2	3	4	5
20	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342	1	2	3	4	5
21	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278	1	2	3	4	5
22	0.9272	0.9625	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212	1	2	3	4	6
23	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143	1	2	3	5	6
24	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070	1	2	4	5	6
25	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996	1	3	4	5	6
26	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918	1	3	4	5	6
27	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838	1	3	4	5	7
28	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755	1	3	4	6	7
29	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669	1	3	4	6	7
30	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581	1	3	4	6	7
31	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490	2	3	5	6	8
32	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396	2	3	5	6	8
33	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300	2	3	5	6	8
34	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202	2	3	5	7	8
35	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100	2	3	5	7	8
36	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997	2	3	5	7	9
37	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891	2	4	5	7	9
38	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782	2	4	5	7	9
39	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672	2	4	6	7	9
40	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559	2	4	6	8	9
41	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443	2	4	6	8	10
42	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325	2	4	6	8	10
43	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206	2	4	6	8	10
44	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083	2	4	6	8	10
45	0.7071	0.7059	0.7046	0,7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959	2	4	6	8	10
46	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833	2	4	6	8	11
47	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704	2	4	6	9	11
48	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574	2	4	7	9	11
49	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441	2	4	7	9	11
50	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307	2	4	7	9	11
51	0.6293	0.6280	0.6266	0.6252	0.6239	0.6255	0.6211	0.6198	0.6184	0.6170	2	5	7	9	11
52	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6404	0.6060	0.6046	0.6032	2	5	7	9	12
53	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892	2	5	7	9	12
54	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750	2	5	7	9	12
55	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606	2	5	7	10	12
56	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461	2	5	7	10	12
57	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314	2	5	7	10	12
58	0.5299	0.5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165	2	5	7	10	12
59	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015	3	5	8	10	13

Natural Cosines

Numbers in different columns to be subtracted**, not added**

°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
60	0.5000	0.4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863	3	5	8	10	13
61	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710	3	5	8	10	13
62	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555	3	5	8	10	13
63	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399	3	5	8	10	13
64	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0.4258	0.4242	3	5	8	11	13
65	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083	3	5	8	11	13
66	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923	3	5	8	11	13
67	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762	3	5	8	11	13
68	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600	3	5	8	11	14
69	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437	3	5	8	11	14
70	0.3420	0.3404	0.3387	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272	3	5	8	11	14
71	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107	3	6	8	11	14
72	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940	3	6	8	11	14
73	0.2924	0.2907	0.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773	3	6	8	11	14
74	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605	3	6	8	11	14
75	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436	3	6	8	11	14
76	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267	3	6	8	11	14
77	0.2250	0.2233	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096	3	6	9	11	14
78	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925	3	6	9	11	14
79	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754	3	6	9	11	14
80	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582	3	6	9	11	14
81	0.1564	0.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409	3	6	9	11	14
82	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236	3	6	9	12	14
83	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063	3	6	9	12	14
84	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889	3	6	9	12	14
85	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715	3	6	9	12	14
86	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541	3	6	9	12	15
87	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366	3	6	9	12	15
88	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192	3	6	9	12	15
89	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017	3	6	9	12	15
90	0.0000														

Fig 2



COSINE CURVE

Quadrant	Angle	cos A =	Examples
First	0 to 90°	cos A	cos 33°26' = 0.8345
Second	90° to 180°	–cos(180° – A)	sin 146°34' = –cos(180° – 146o 34') = –cos 33o26' = –0.8345
Third	180° to 270°	cos(A – 180°)	cos 213°26' = –cos(213°26' – 180°) = –cos 33°26' = –0.8345
Fourth	270° to 360°	cos(360° – A)	cos 326°34' = cos(360° – 326°34') = cos 33°26' = 0.8345

Natural Tangents

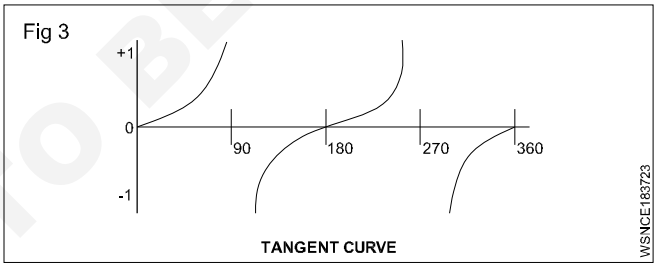
°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	3	6	9	12	15
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	3	6	9	12	15
2	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507	3	6	9	12	15
3	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682	3	6	9	12	15
4	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857	3	6	9	12	15
5	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033	3	6	9	12	15
6	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210	3	6	9	12	15
7	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388	3	6	9	12	15
8	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566	3	6	9	12	15
9	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745	3	6	9	12	15
10	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926	3	6	9	12	15
11	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107	3	6	9	12	15
12	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290	3	6	9	12	15
13	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475	3	6	9	12	15
14	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661	3	6	9	12	16
15	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849	3	6	9	13	16
16	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038	3	6	9	13	16
17	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230	3	6	10	13	16
18	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424	3	6	10	13	16
19	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620	3	7	10	13	16
20	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819	3	7	10	13	17
21	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020	3	7	10	13	17
22	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224	3	7	10	14	17
23	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431	3	7	10	14	17
24	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642	4	7	11	14	18
25	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856	4	7	11	14	18
26	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073	4	7	11	15	18
27	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295	4	7	11	15	18
28	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520	4	8	11	15	19
29	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750	4	8	12	15	19
30	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985	4	8	12	16	20
31	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224	4	8	12	16	20
32	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469	4	8	12	16	20
33	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720	4	8	13	17	21
34	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976	4	9	13	17	21
35	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239	4	9	13	17	22
36	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508	5	9	14	18	23
37	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785	5	9	14	18	23
38	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069	5	9	14	19	24
39	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361	5	10	15	20	24
40	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662	5	10	15	20	25
41	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972	5	10	16	21	26
42	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293	5	11	16	21	27
43	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623	6	11	17	22	28
44	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965	6	11	17	23	28
45	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319	6	12	18	24	30
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686	6	12	18	25	31
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067	6	13	19	25	32
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463	7	13	20	27	33
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875	7	14	21	28	34
50	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305	7	14	22	29	36
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753	8	15	23	30	38
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222	8	16	24	31	39
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713	8	16	25	33	41
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229	9	17	26	34	43
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770	9	18	27	36	45
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340	10	19	29	38	48
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941	10	20	30	40	50
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577	11	21	32	43	53
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251	11	23	34	45	56

Natural Tangents

°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
60	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966	12	24	36	48	60
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728	13	26	38	51	64
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542	14	27	41	55	68
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413	15	29	44	58	73
64	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348	16	31	47	63	78
65	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355	17	34	51	68	85
66	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445	18	37	55	73	92
67	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627	20	40	60	79	99
68	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916	22	43	65	87	108
69	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326	24	47	71	95	119
70	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878	26	52	78	104	131
71	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595	29	58	87	116	145
72	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506	32	64	96	129	161
73	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646	36	72	108	144	180
74	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062	41	81	122	163	204
75	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812	46	93	139	186	232
76	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972	53	107	160	213	267
77	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646					
78	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970					
79	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140					
80	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432					
81	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264					
82	7.1154	7.2066	7.3002	6.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285					
83	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572					
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20					
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95					
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27					
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0					
90	×														

DIFFERENCES UNTRUSTWORTHY HERE

Fig 3



TANGENT CURVE

Quadrant	Angle	tan A =	Examples
First	0 to 90°	tan A	tan 56°17' = 1.4986
Second	90° to 180°	–tan(180° – A)	tan 123°43' = –tan(180° – 123° 43') = –tan 56°17' = –1.4986
Third	180° to 270°	tan(A – 180°)	tan 236°17' = tan(236°17' – 180°) = tan 56°17' = 1.4986
Fourth	270° to 360°	–tan(360° – A)	tan 303°43' = –tan(360° – 303°43') = – tan 56°17' = –1.4986

Trigonometry - Application in calculating height and distance (Simple applications)

Heights and Distances

One of the practical applications of Trigonometry is to find distances and heights of distant and inaccessible objects. Two angles are often used in the practical applications of Trigonometry and they are defined as follows:

The angle between a horizontal plane through an observer's eye and line joining the eye to an object is called

i **The angle of elevation** when the object is higher than the eye. (Fig 1)

ii **The angle of depression** when the object is lower than the eye. (Fig 2)

Fig 1

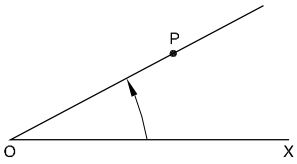


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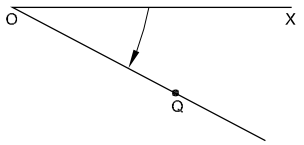
Fig 2



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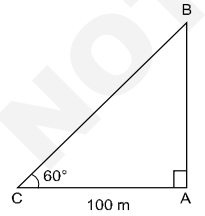
If OX be a horizontal line through 'O' the observer's eye and 'P' any point above OX, then XOP is the angle of elevation of P at O. If 'Q' below OX, then XOQ is called the angle of depression of Q at O.

Example

1 From a place of 100m away from the foot of a tower the top is seen at an angle of elevation 60°. Find the height of the tower.

Fig 3



WSNCE183813

Given: AB = Tower

AC = Distance between Tower and Point of Elevation

Angle of Elevation $\angle ACB = 60°$

To Find: Height of Tower AB = ?

In Right Angled Triangle ABC

$$\text{Tan}\,\theta = \frac{AB}{AC}$$

$$AB = \text{Tan}\,\theta \times AC$$

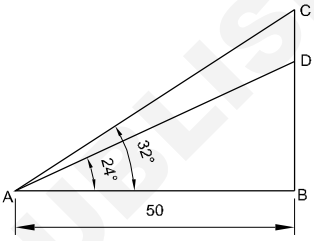
$$= \text{Tan}\,60° \times 100$$

$$= 1.7321 \times 100 = 173.21 \text{ m}$$

Height of Tower = 173.21 m

2 A flag pole stands on the top of a building when viewed from a distance of 50 m. (measured horizontally) the angle of elevation of the top and bottom of the flag staff are 24 and 32° respectively. Find the height of the flag pole.

Fig 4



WSNCE183814

Let CD be the flag pole, 'A' the point of observation and 'B' a point on the same level as 'A' and directly underneath the flag pole.

In the triangle DAB

$$\tan 24^\circ = \frac{BD}{AB} = \frac{BD}{50}$$

$$\therefore BD = 50.\tan 24^\circ$$

$$= 50 \times 0.4452$$

$$= 22.26 \text{ m}$$

In the triangle CAB,

$$\tan 32^\circ = \frac{CB}{AB} = \frac{BC}{50}$$

$$\therefore BC = 50 \,.\, \tan 32^\circ$$

$$= 50 \times 0.6249$$

$$= 31.25$$

$$\text{Hence } DC = 31.25 - 22.26$$

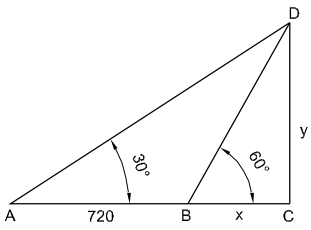
$$= 8.99$$

$$= 9 \text{ m nearly.}$$

3 At a certain point on the ground, it is found that the angle of elevation of the top of a tower is 30°. On walking 720 m. towards the foot of tower, the angle of elevation is found to be 60°. Find the height of the tower.

Let CD be the tower and A and B be the points from which the tower is observed and let BC be x and CD be y.

Fig 5



WSNCE183815

In triangle ADC,

$$\tan 30^\circ = \frac{CD}{AC} = \frac{y}{x+720}$$

$$y = (x + 720) \tan 30^0$$

$$\text{Angle BDC, } \tan 60^\circ = \frac{CD}{BC} = \frac{y}{x}$$

$$\therefore y = x \tan 60^\circ$$

$$\text{Hence, } (x + 720) \tan 30^\circ = x \tan 60^\circ$$

$$(x+720)\frac{1}{\sqrt{3}} = x\sqrt{3}$$

$$\text{i.e.} \frac{x+720}{\sqrt{3}} = \sqrt{3}\ x$$

$$\therefore x + 720 = 3x$$

$$720 = 3x - x$$

$$2x = 720$$

$$x = \frac{720}{2} = 360$$

$$\text{Hence } y = 360 \tan 60^\circ$$

$$= 360\sqrt{3}$$

$$\tan 60^0 = \frac{y}{x}$$

$$\tan 60^0 = \frac{y}{360}$$

$$y = \tan 60^0 \times 360$$

Hence the height of the tower is $360\sqrt{3}$

$$= 360 \times 1.732$$

Height of the tower = 623.5 metres

4 A divider having legs of equal length of 10 cm is opened so that its points are 4.5 cm apart. Using trigonometrical tables. Find the angle between the legs.

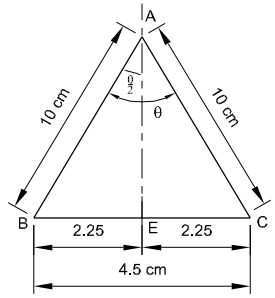
Distance between the two legs of divider = BC = 4.5 cm

AC = AB ; Length of leg = 10 cm

The perpendicular line drawn from the centre of BC, (point E) to point A makes two right angled triangle ABE and AEC. If the angle between two legs of divider is θ

$$\angle BAE = \frac{\theta}{2}$$

Fig 6



WSNCE183816

$$= Sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\text{Sin}\frac{\theta}{2} = \frac{BE}{AB}$$

$$= \frac{2.25}{10} = 0.225$$

Find the **θ** value of 0.225 from sin table

$$0.225 = 13^0$$

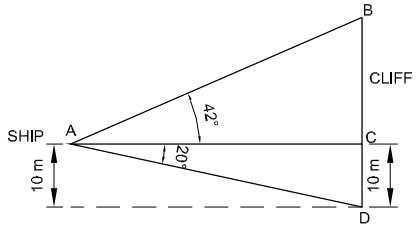
$$\frac{\theta}{2} = 13^0$$

$$\theta = 13^0 \times 2 = 26^\circ$$

Angle between two leg of divider is 26°

5 A man on the deck of a ship is 10 m above water level. He observes that angle of elevation of a cliff is 42° and angle of depression of its base is 20°. Calculate (i) the distance of the cliff from the ship. (ii) the height of the cliff.

Fig 7



WSNCE183817

Find the distance between ship and cliff

From right angled triangle CAD

$$tan\ \theta = \frac{\text{Opposite side}}{\text{Adjacent Side}} = \frac{CD}{AC}$$

$$\tan 20^\circ = \frac{10}{AC}$$

$$0.3640 = \frac{10}{AC}$$

$$AC = \frac{10}{0.3640} = 27.473\,m$$

Height of cliff BD = DC + CB

Find CB in right angled triangle BAC

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent Side}} = \frac{BC}{AC}$$

$$\tan 42^\circ = \frac{BC}{27.473}$$

$$0.9004 = \frac{BC}{27.473}$$

BC = 0.9004 x 27.473

= 24.737 m

Height of cliff BD = DC + CB

= 10 + 24.737

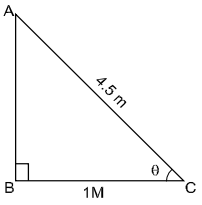
= 34.737 m

Distance between ship and cliff = 27.473 m

Height of cliff = 34.737 m

6 The foot of a 4.5 m long ladder is placed at 1 m away from the wall. Find the angle which the ladder makes with the ground

Fig 8



In Right angled Δ

$$\text{Cos C} = \frac{BC}{AC}$$

$$\text{Cos}\,\theta = \frac{1\text{ m}}{4.5\text{ m}}$$

cos θ = 0.2222

0.2233 = cos 77° 6'

(-) 0.0011 = 4' (+)

0.2222 = cos 77° 10'

θ = 77° 10'

Ladder makes the angle = 77° 10'

7 A Line man is working on a road places his ladder is 12 m in length at a point on the road such that it makes an angle of 60⁰ with the ground, when it is placed against a lamp placed against another lamp post directly on the opposite side of the road it makes an angle of 30⁰. Find the distance between the 2 lamp posts.

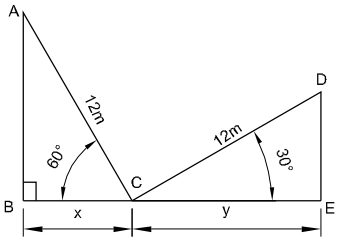
AC and CD are Ladder

AB and DE are lamp post

In Δ ABC

$$\text{Cos}\,60^\circ = \frac{BC}{AC} = \frac{x}{12\text{ m}}$$

Fig 9



$\therefore x$ = cos 60° x 12 m

= 0.5000 x 12 m

= 6 m

In Δ CDE

$$\text{Cos}\,30^\circ = \frac{CE}{CD} = \frac{y}{12\text{ m}}$$

$\therefore y$ = cos 30° x 12 m

= 0.8660 x 12 m

= 10.392 m

Distance between two lamp posts

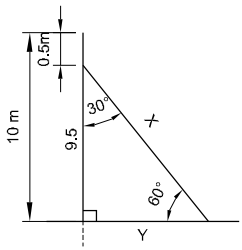
= x + y

= 6 + 10.392 m

Distance between two lamp posts = 16.392 m

8 A pole stands 10 metre above the ground and stay wire is fixed to the pole at 0.5 metre from the top. If the stay wire is to make an angle of 60⁰ with the horizontal, find the distance of stay rod from the base of the pole. Also find the length of the stay wire.

Fig 10



stay wire fixed 0.5m below from the top of the pole

$$\text{Sin}\,\theta = \frac{OPP}{HYP}$$

$$\text{Sin}\,60^\circ = \frac{9.5}{x}$$

x x sin 60° = 9.5 m

$$x = \frac{9.5}{\text{Sin}\,60^\circ}$$

$$= \frac{9.5}{0.8660} = 10.9699$$

Stay wire (x) = 10.97 m

$$\tan\theta = \frac{Opp}{Adj}$$

$$\tan 60^{o} = \frac{9.5}{y}$$

$$y \tan 60^{o} = 9.5$$

$$y = \frac{9.5}{\tan 60^{\circ}}$$

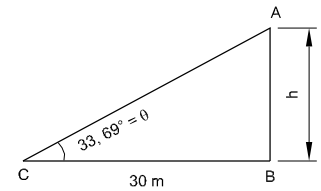
$$= \frac{9.5}{1.7321} = 5.48 \text{ m}$$

Length of the stay wire = 10.97 metre

distance of the stay wire from the base of the pole = 5.48 metre

9 An electrician standing 30metres away from the base of a transmission line tower looks at the top of the tower. His line of sight makes an angle of 33.69^{0} with the horizontal. Find the height of the tower?

Fig 11



WSNCE18381B

Given:

Distance between the Electrician and the tower } = BC = 30m

Height of the tower AB (h) = ?

$33.69^{0} = 33^{0} + (0.69 \times 60')$ ($\therefore$ 1° = 60')

$= 33^{0} + 41'$

$33.69^{0} = 33^{0}41'$

In Right angled triangle ABC

$$\text{Tan}\,\theta = \frac{AB}{BC}$$

$AB = \tan\theta \times BC$ ($\therefore \tan 33°41' = \text{Natural tan} = 0.6665$)

$= \tan 33°41' \times 30$

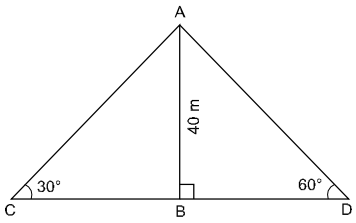
$= 0.6665 \times 30$ m

$= 19.995$ m

Height of the tower = 19.995 m

10 Two persons standing at a distance on the opposite side of tower see the top of a tower at an angle of 30° and 60° respectively. The height of the tower is 40meters. Find the distance between the two persons.

Fig 12



WSNCE18381C

Given: Height of the Tower AB = 40m

Say, Two persons standing opposite side of tower at the point of C and D.

$\angle ACB = 30°$

$\angle ADB = 60°$

To Find: Distance between the Two Persons CD = ?

In Right Angled Triangle ABC

$$\text{Tan}\,30° = \frac{AB}{CB}$$

$$CB = \frac{AB}{\text{Tan}\,30°} = \frac{40}{0.5774} = 69.28$$

$CB = 69.28$m

In Right Angled Triangle ABD

$$\text{Tan}\,60° = \frac{AB}{BD}$$

$$BD = \frac{AB}{\text{Tan}\,60°} = \frac{40}{1.7321} = 23.1$$

$BD = 23.1$m

Distance between the Two persons = CD

$CD = CB + BD$

$= 69.28 + 23.1$

$CD = 92.38$m

Distance between the two persons (CD) = 92.38m

Assignment

1 Two men are on the opposite sides of the tower. They measure the angle of elevation of the top of the tower is 45° and 30° respectively. If the height of the tower is 80 meters. Find the distance of the men?

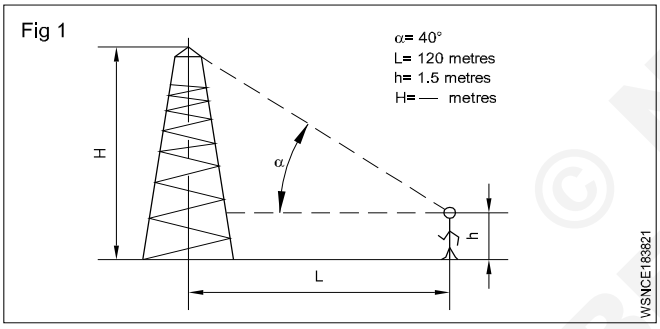
2 A shadow of an electric pole is reduced by 4 metres when the sun changes its angle of elevation from 30° to 45°. If the pole is buried in the ground by 2 metres, find the total length of the pole?

3 The angle of elevation of the top on an unfinished tower at a point distance 120 metre from its base is 45°. How much higher must the tower be raised so that its angle of elevation at the same may be 60°.

4 Two objects on horizontal plane in the same line of the foot of the cliff from with the top of cliff angles of elevation of 30° and 45°. If the height of the cliff is 100 m, calculate the distance between the two objects.

5 From a point 'A' on the ground at unknown distance from the base of the radio tower, the angle of elevation of the top of mast is 65°. Proceeding in the same straight line to the point 'B' 50 m from 'A', the angle of elevation is reduced to 50°. Find the height of the mast and distance of 'A' from the mast.

6



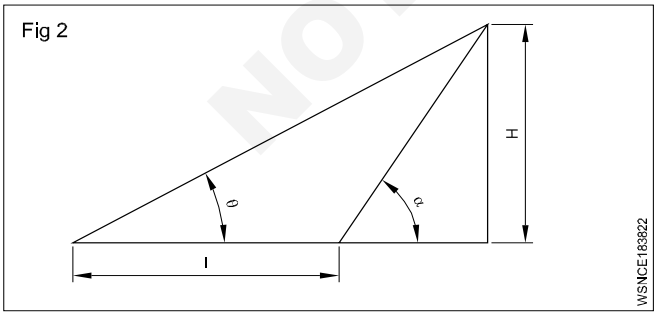
α = 40°

L = 120 metres

h = 1.5 metres

H = _____ metre

7



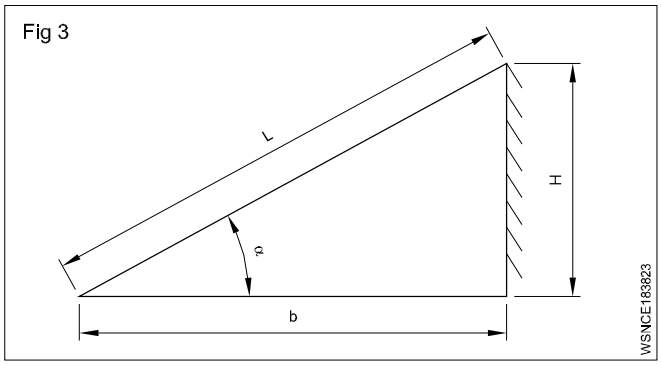
θ = 25°

α = 50°

l = 100 metres

H = _____ metre

8



α = 75°

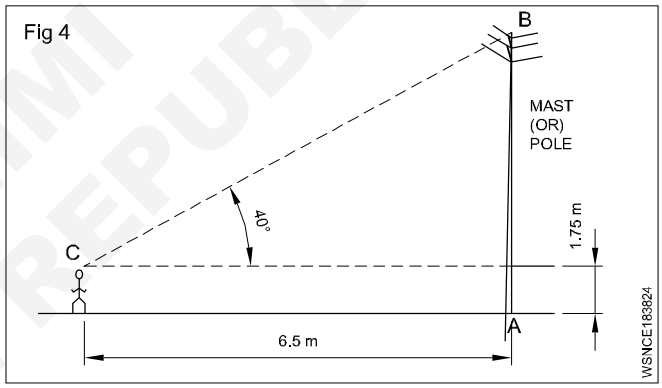
b = 3 metres

H = _____ metres

L = _____ metres

9 The height of the pole,

AB = _____ m.



10 Find the values of the given angles.

1 Sin 65°

2 Sin 42° 23'

3 Sin 66° 35' 32"

4 Cos 47° 39'

5 Tan 28° 45'

11 Find corresponding angles for given values.

1 Sin θ = 0.3062

2 Sin θ = 0.04802

3 Cos θ = 0.6446

4 Tan θ = 0.3411

6 Tan θ = 2.3868

12 The slant height of a cone is 12.25 cm and the vertex angle is 110^0. Calculate its base.

13 A ladder 2.5 m long makes an angle of 60^0 with the ground. Find the height of the wall where the ladder touches the wall.